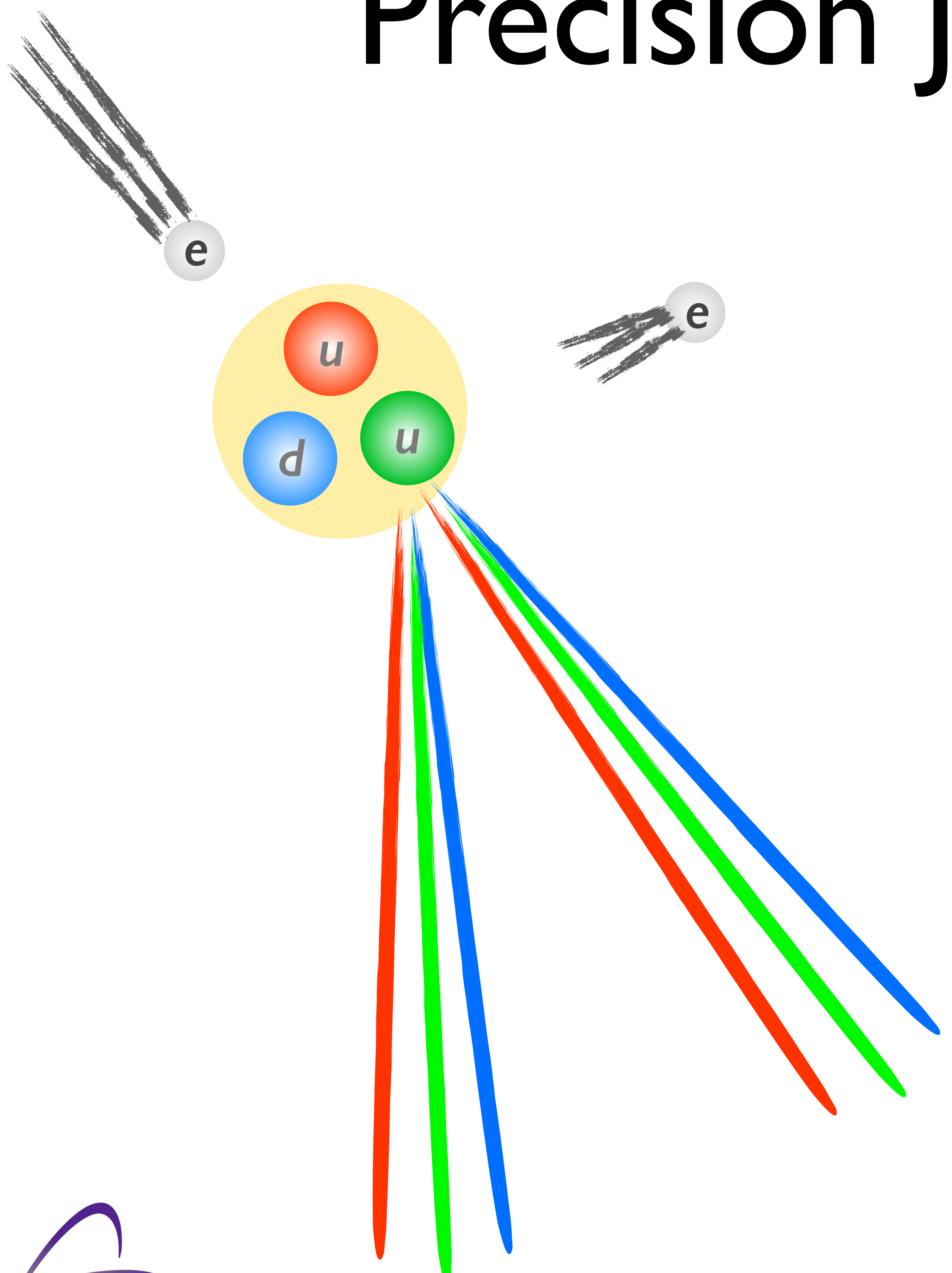
Precision Jet Physics in DIS



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in collaboration with
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and **Iain Stewart** *(MIT)*

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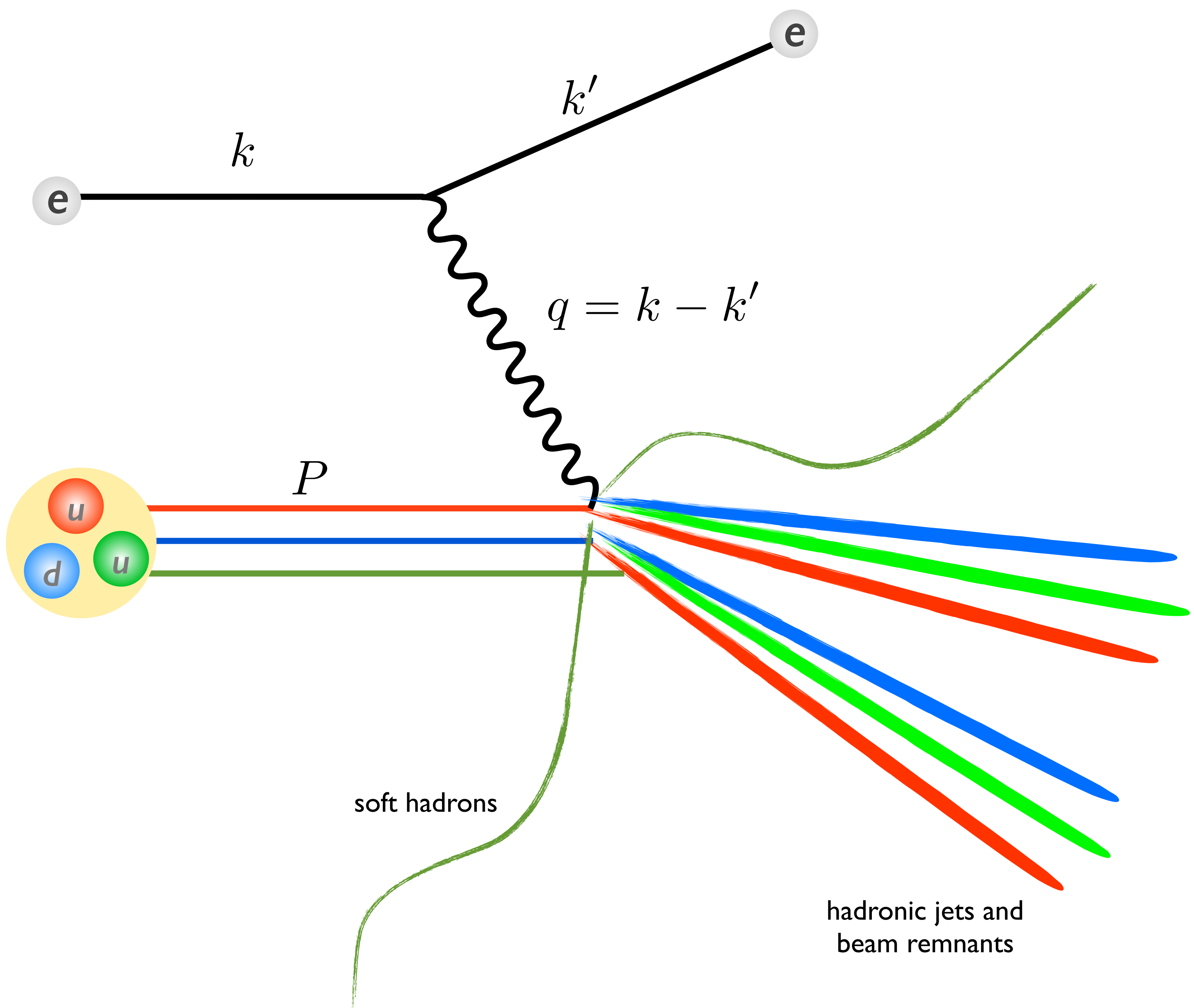
Los Alamos
NATIONAL LABORATORY

Outline

- Jets, Event Shapes, and the Strong Coupling
- Defining 1-Jettiness in DIS
- Factorization for 1-Jettiness in SCET
- Universal Nonperturbative Effects
  in DIS 1-Jettiness
- N^3LL Resummed Predictions for DIS 1-Jettiness
- DIS event shapes in current and future data

DIS Kinematics



$$s = (k+P)^2$$ squared center-of-mass energy

$$Q^2 = -q^2$$ momentum transfer

$$x = \frac{Q^2}{2P\cdot q}$$ Björken scaling variable

$$y = \frac{P\cdot q}{P\cdot k}$$ lepton energy loss in proton rest frame

$$Q^2 = xys$$

$$p_X = q + P$$ total momentum of final hadronic state

$$p_X^2 = \frac{1-x}{x}Q^2$$ invariant mass of final hadronic state

Limit $x \to 1$ corresponds to single collimated jet in final state

We will look away from *x* = *1* at *two*-jet like final states

Strong Coupling from Jets in DIS

Process	Collab.	Value	Exp.	Th.	Total	(%)
(1) Inc. jets at low Q^2	H1	0.1180	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.6}_{-8.1}$
(2) Dijets at low Q^2	H1	0.1155	0.0018	$^{+0.0124}_{-0.0093}$	$^{+0.0125}_{-0.0095}$	$^{+10.8}_{-8.2}$
(3) Trijets at low Q^2	H1	0.1170	0.0017	$^{+0.0091}_{-0.0073}$	$^{+0.0093}_{-0.0075}$	$^{+7.9}_{-6.4}$
(4) Combined low Q^2	H1	0.1160	0.0014	$^{+0.0094}_{-0.0079}$	$^{+0.0095}_{-0.0080}$	$^{+8.2}_{-6.9}$
(5) Trijet/dijet at low Q^2	H1	0.1215	0.0032	$^{+0.0067}_{-0.0059}$	$^{+0.0074}_{-0.0067}$	$^{+6.1}_{-5.5}$
(6) Inc. jets at medium Q^2	H1	0.1195	0.0010	$^{+0.0052}_{-0.0040}$	$^{+0.0053}_{-0.0041}$	$^{+4.4}_{-3.4}$
(7) Dijets at medium Q^2	H1	0.1155	0.0009	$^{+0.0045}_{-0.0035}$	$^{+0.0046}_{-0.0036}$	$^{+4.0}_{-3.1}$
(8) Trijets at medium Q^2	H1	0.1172	0.0013	$^{+0.0053}_{-0.0032}$	$^{+0.0055}_{-0.0035}$	$^{+4.7}_{-3.0}$
(9) Combined medium Q^2	H1	0.1168	0.0007	$^{+0.0049}_{-0.0034}$	$^{+0.0049}_{-0.0035}$	$^{+4.2}_{-3.0}$
(10) Inc. jets at high Q^2 (anti-k_T)	ZEUS	0.1188	$^{+0.0036}_{-0.0035}$	$^{+0.0022}_{-0.0022}$	$^{+0.0042}_{-0.0041}$	$^{+3.5}_{-3.5}$
(11) Inc. jets at high Q^2 (SIScone)	ZEUS	0.1186	$^{+0.0036}_{-0.0035}$	$^{+0.0025}_{-0.0025}$	$^{+0.0044}_{-0.0043}$	$^{+3.7}_{-3.6}$
(12) Inc. jets at high Q^2 (k_T; HERA I)	ZEUS	0.1207	$^{+0.0038}_{-0.0036}$	$^{+0.0022}_{-0.0023}$	$^{+0.0044}_{-0.0043}$	$^{+3.6}_{-3.6}$
(13) Inc. jets at high Q^2 (k_T; HERA II)	ZEUS	0.1208	$^{+0.0037}_{-0.0032}$	$^{+0.0022}_{-0.0022}$	$^{+0.0043}_{-0.0039}$	$^{+3.6}_{-3.2}$
(14) Inc. jets in γp (anti-k_T)	ZEUS	0.1200	$^{+0.0024}_{-0.0023}$	$^{+0.0043}_{-0.0032}$	$^{+0.0049}_{-0.0039}$	$^{+4.1}_{-3.3}$
(15) Inc. jets in γp (SIScone)	ZEUS	0.1199	$^{+0.0022}_{-0.0022}$	$^{+0.0047}_{-0.0042}$	$^{+0.0052}_{-0.0047}$	$^{+4.3}_{-3.9}$
(16) Inc. jets in γp (k_T)	ZEUS	0.1208	$^{+0.0024}_{-0.0023}$	$^{+0.0044}_{-0.0033}$	$^{+0.0050}_{-0.0040}$	$^{+4.1}_{-3.3}$
(17) Jet shape	ZEUS	0.1176	$^{+0.0013}_{-0.0028}$	$^{+0.0091}_{-0.0072}$	$^{+0.0092}_{-0.0077}$	$^{+7.8}_{-6.5}$
(18) Subjet multiplicity	ZEUS	0.1187	$^{+0.0029}_{-0.0019}$	$^{+0.0093}_{-0.0076}$	$^{+0.0097}_{-0.0078}$	$^{+8.2}_{-6.6}$
HERA average 2004		0.1186	±0.0011	±0.0050	±0.0051	±4.3
HERA average 2007		0.1198	±0.0019	±0.0026	±0.0032	±2.7

Table 1: Values of $\alpha_s(M_Z)$ extracted from jet observables at HERA together with their uncertainties (rows 1 to 18). The 2004 [10] and 2007 [11] HERA averages are shown in the last two rows.

Extractions from exclusive jet cross sections have order 10% uncertainty, dominated by theory

Improve to level of e^+e^-?

C. Glasman, in the Proceedings of the Workshop on Precision Measurements of α_s [1110.0016]

Challenges to Precision Jet Cross Sections

- Jet cross sections typically depend on
  - choice of jet algorithm
  - jet sizes
  - jet vetoes (for exclusive jet cross sections)
- These parameters generate a number of logarithms (non-global logs, logs of radii R, etc.) in perturbation theory which are challenging to resum *(NB: very recent progress!)*

  e.g. Larkoski, Moult, Neill (2015-16); Chien, Hornig, CL (2015)
- ***N*-Jettiness**: a *global* observable picking out *N*-jet final states by measurement of a *single* parameter, logs of which *can* be resummed in perturbation theory by standard RGE

e^+e^- Thrust: high precision extraction of α_s

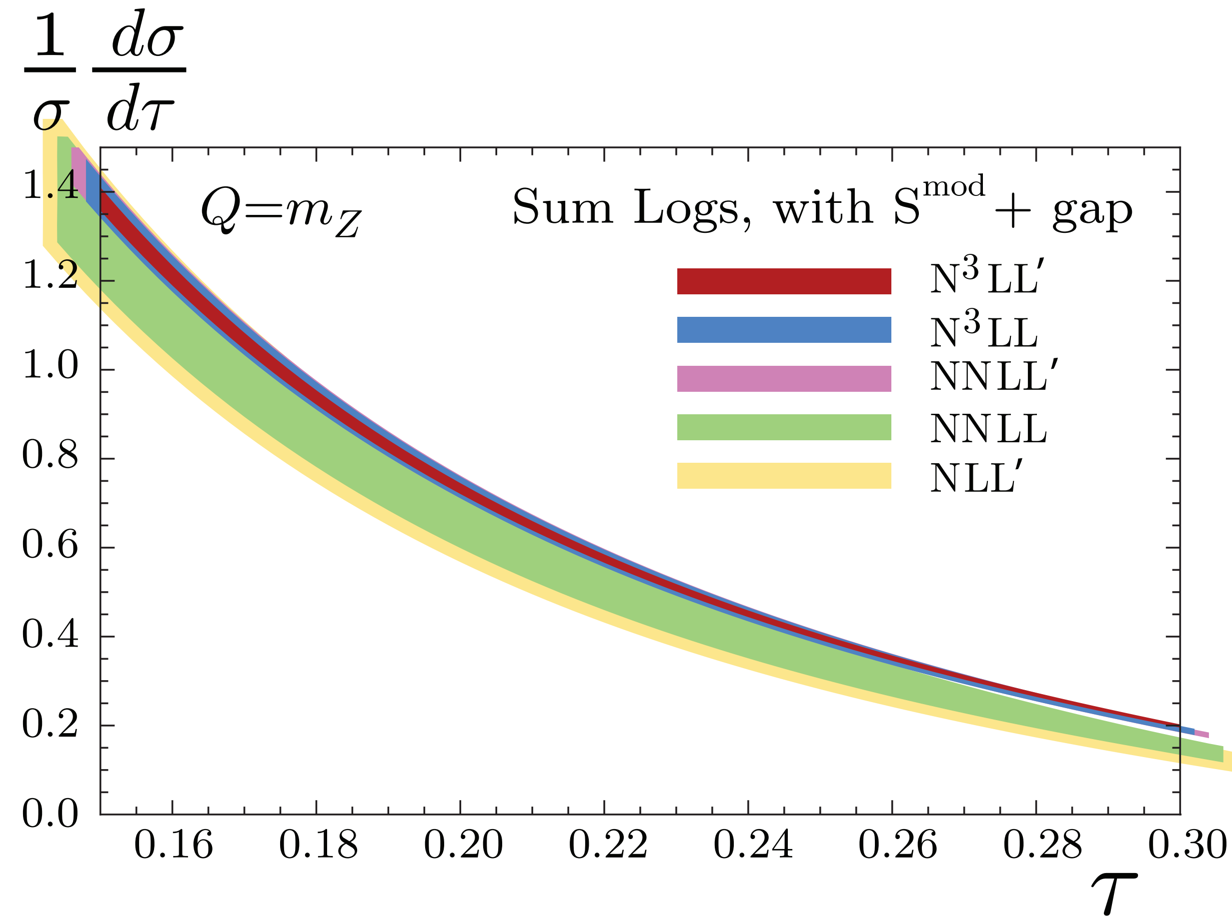
(2-jettiness)

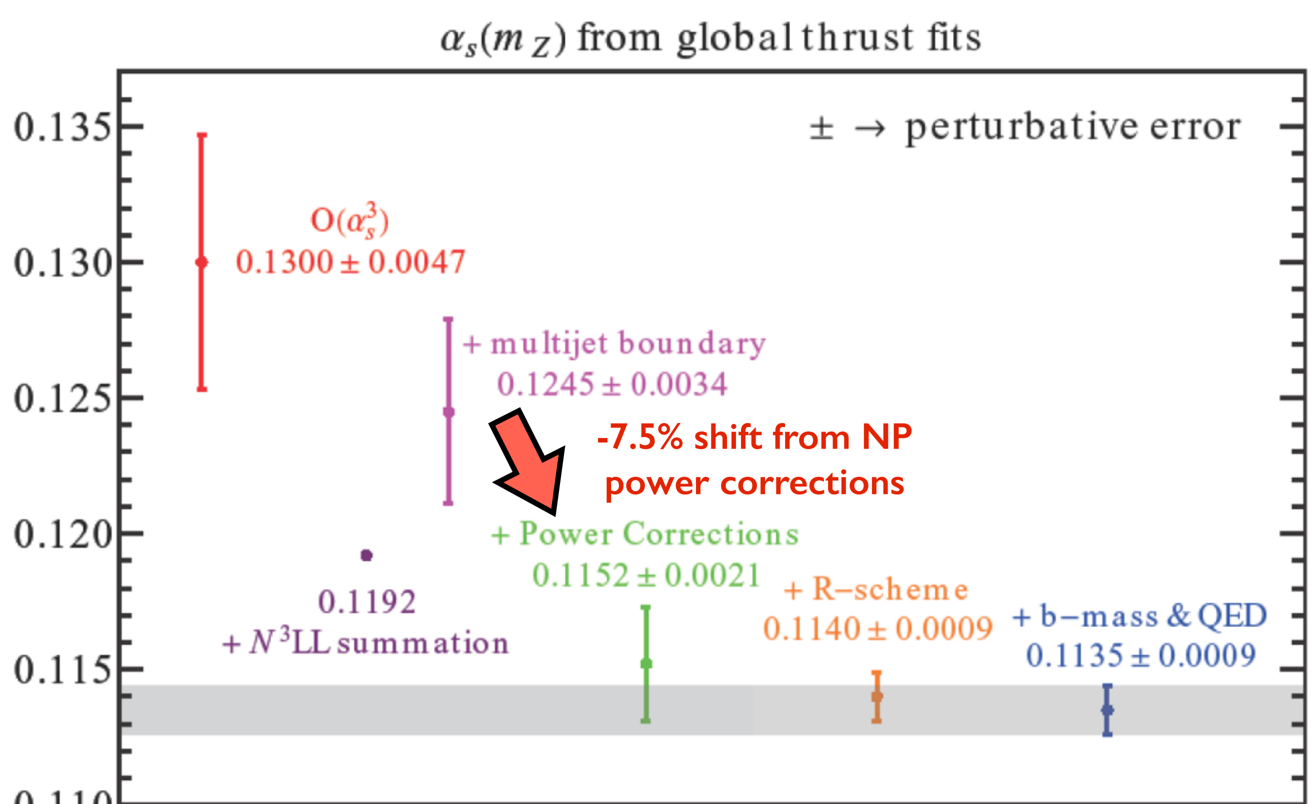
NNNLL perturbative prediction + nonperturbative soft power correction led to most precise extraction of strong coupling from event shapes

Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

NNNLL resummed perturbative distribution

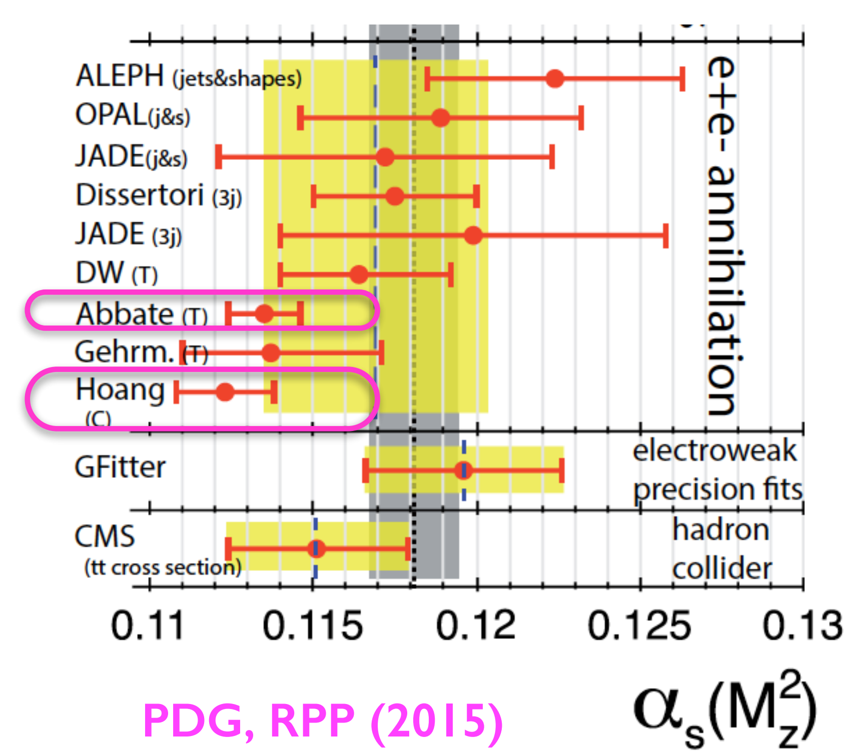
Becher, Schwartz (2008)





Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

recent SCET-based extractions



PDG, RPP (2015)

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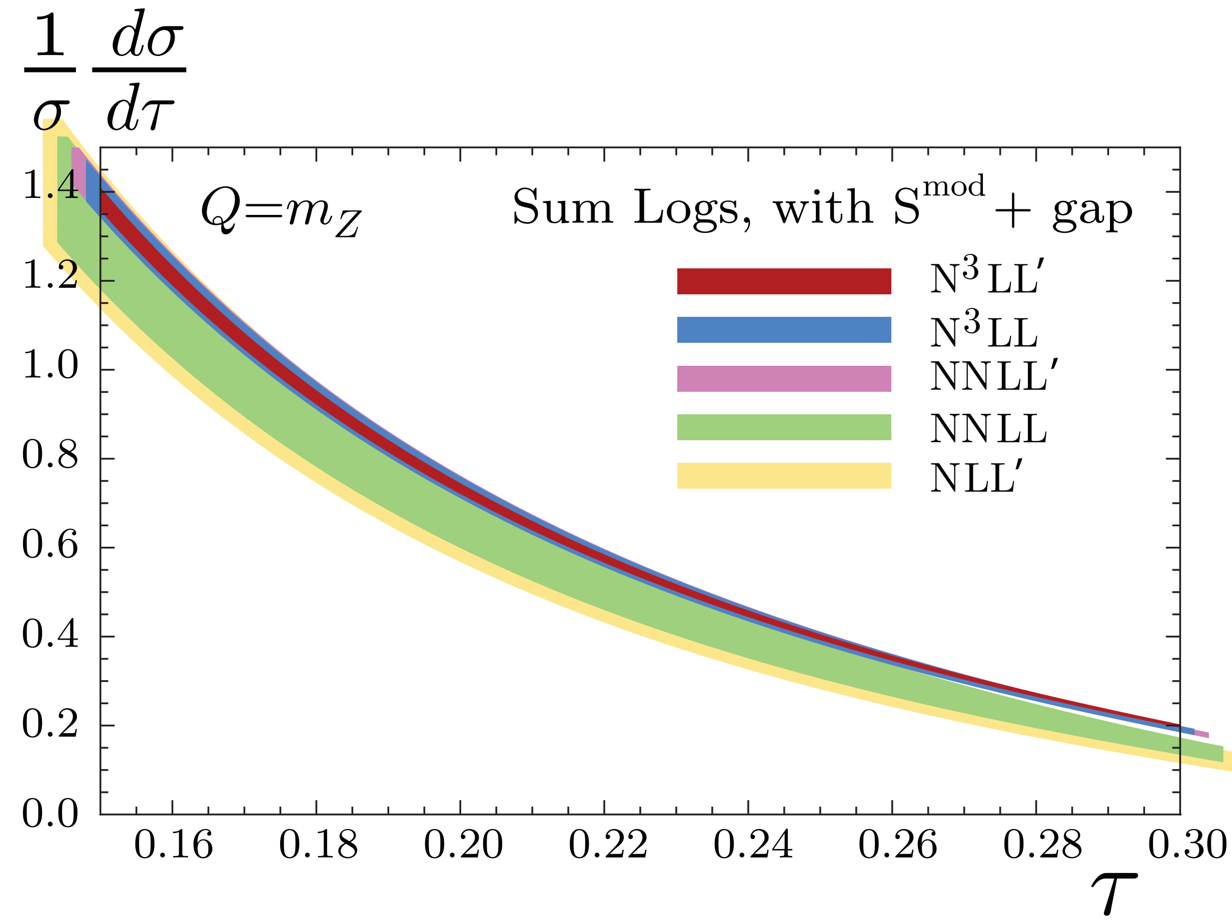

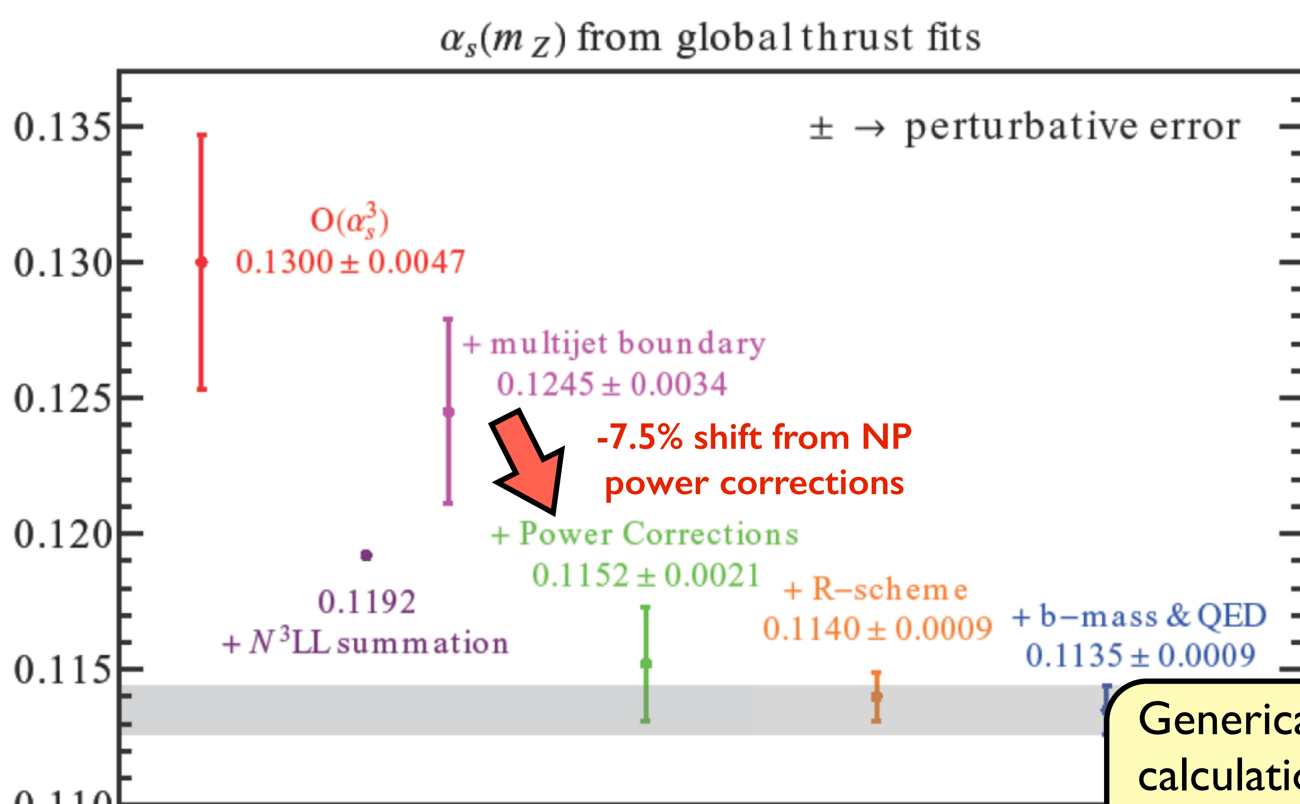

Abbate, Fickinger, Hoang, Mateu, Ste

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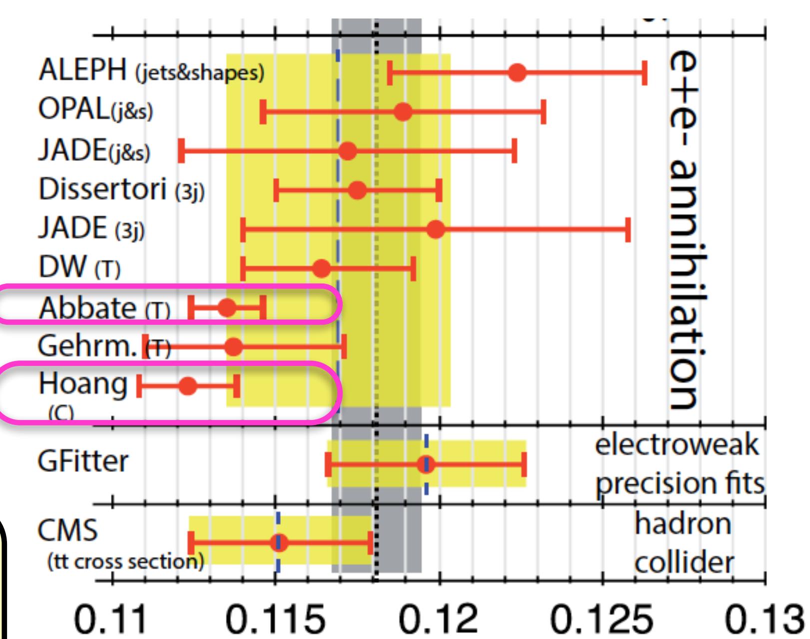


PDG, RPP (2015)

$\alpha_s(M_Z^2)$

Generically, better perturbative calculations + rigorous treatment of nonperturbative corrections gives smaller α_s

N-jettiness

An *inclusive* event shape over all final state hadrons *excluding* more than *N* jets:

$$\tau_N = \frac{2}{Q^2}\sum_i \min\{q_B\cdot p_i, q_1\cdot p_i, \ldots, q_N\cdot p_i\}$$

Stewart, Tackmann, Waalewijn (2010)

Vector q_B is aligned with the incoming proton beam and $q_{1,\ldots,N}$ with final state jets.
Final state hadrons *i* are grouped with the axis "closest" to it.

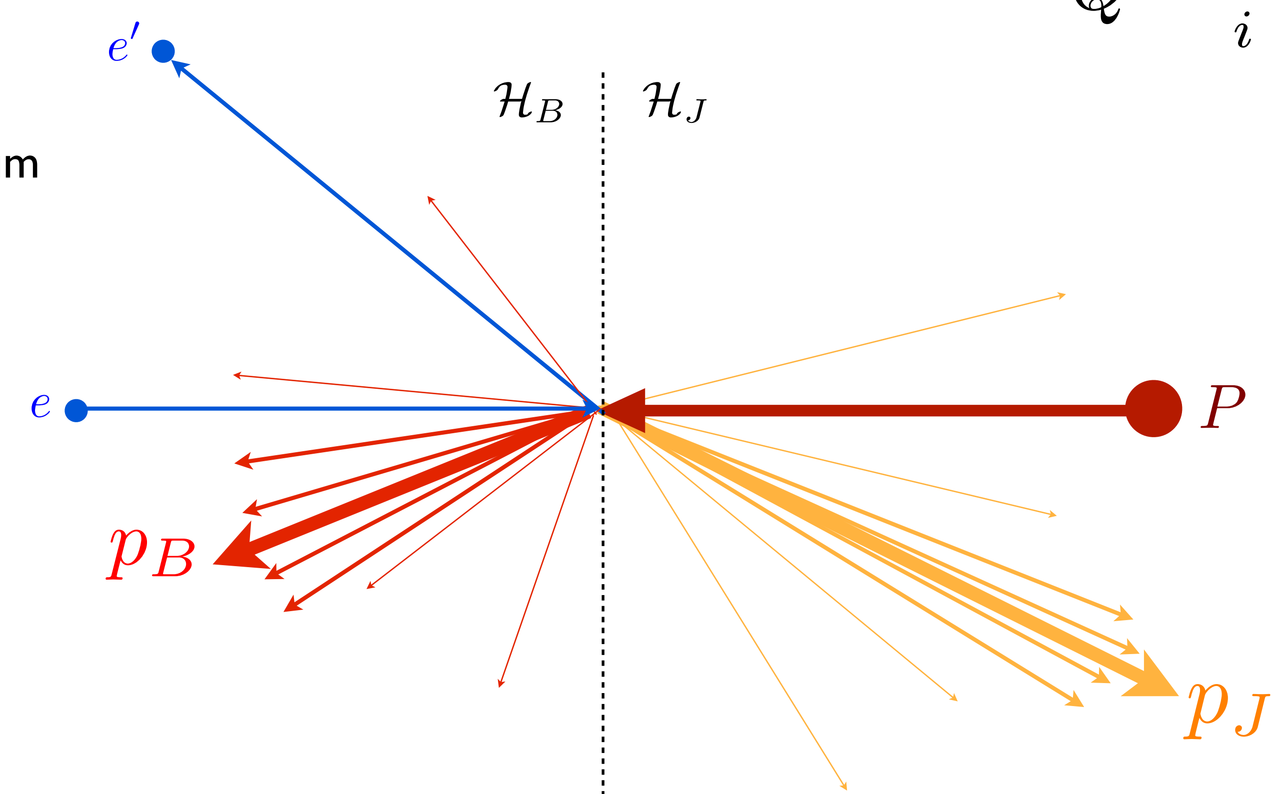
As $\tau_N \to 0$, final state contains exactly *N+1* pencil-like jets (one from beam radiation).

We will look at "1-jettiness" in DIS.

$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B\cdot p_i, q_J\cdot p_i\}$$

Center-of-momentum frame:

$$k = \sqrt{s}\frac{n_z}{2}$$



$$P = \sqrt{s}\frac{\bar{n}_z}{2}$$

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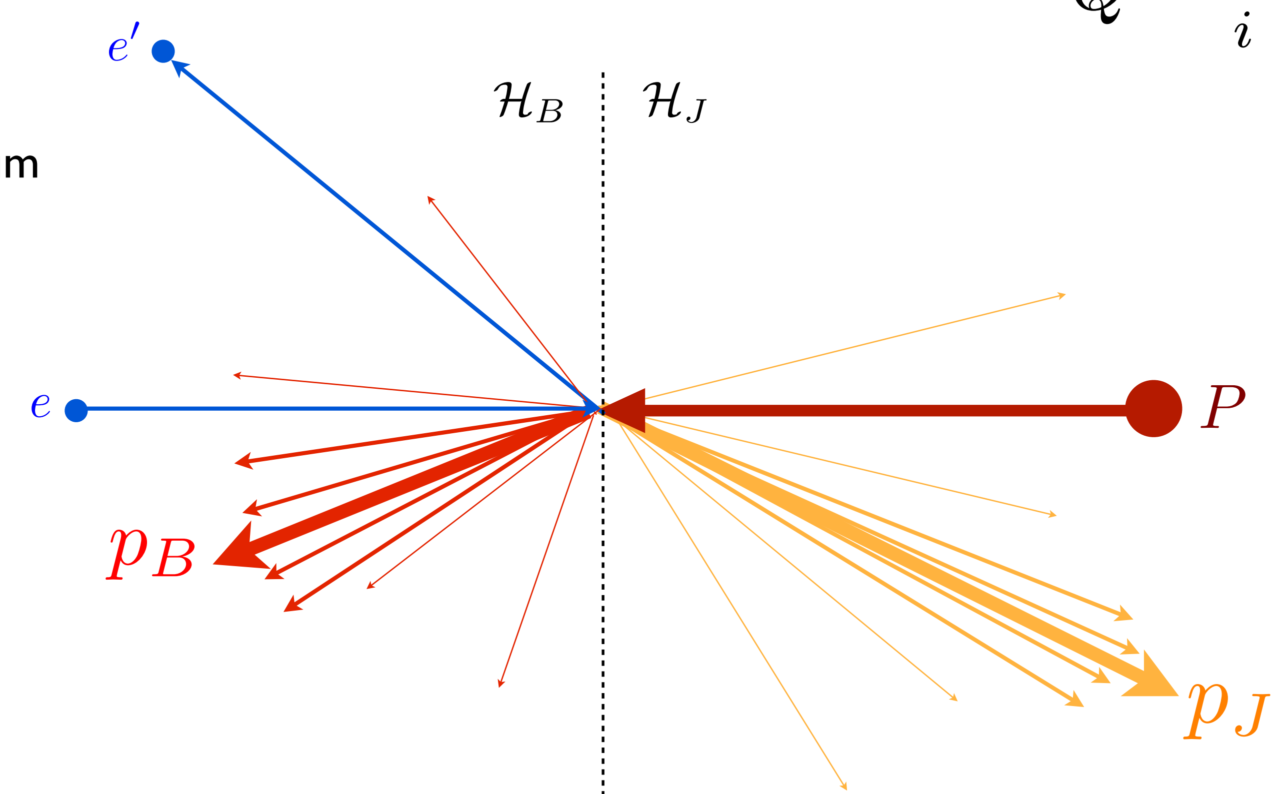
We will look at "1-jettiness" in DIS.

$$\tau_1 = \frac{2}{Q^2}\sum_i \min\{q_B \cdot p_i, q_J \cdot p_i\}$$

How shall we pick q_B and q_J?

Center-of-momentum frame:

$k = \sqrt{s}\frac{n_z}{2}$

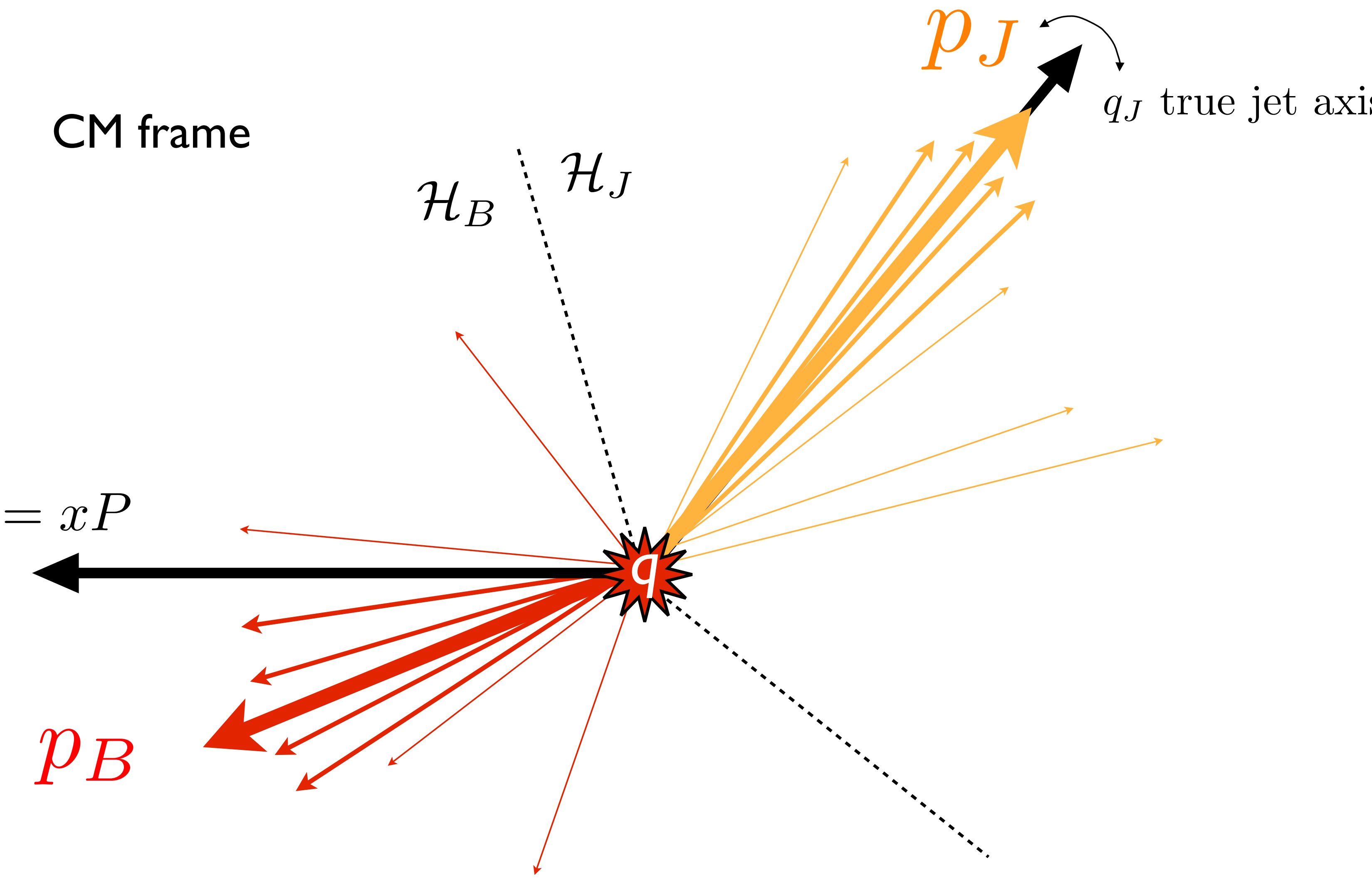


$P = \sqrt{s}\frac{\bar{n}_z}{2}$

Choices for DIS 1-jettiness

D. Kang, CL, Stewart (2013)

τ_1^a



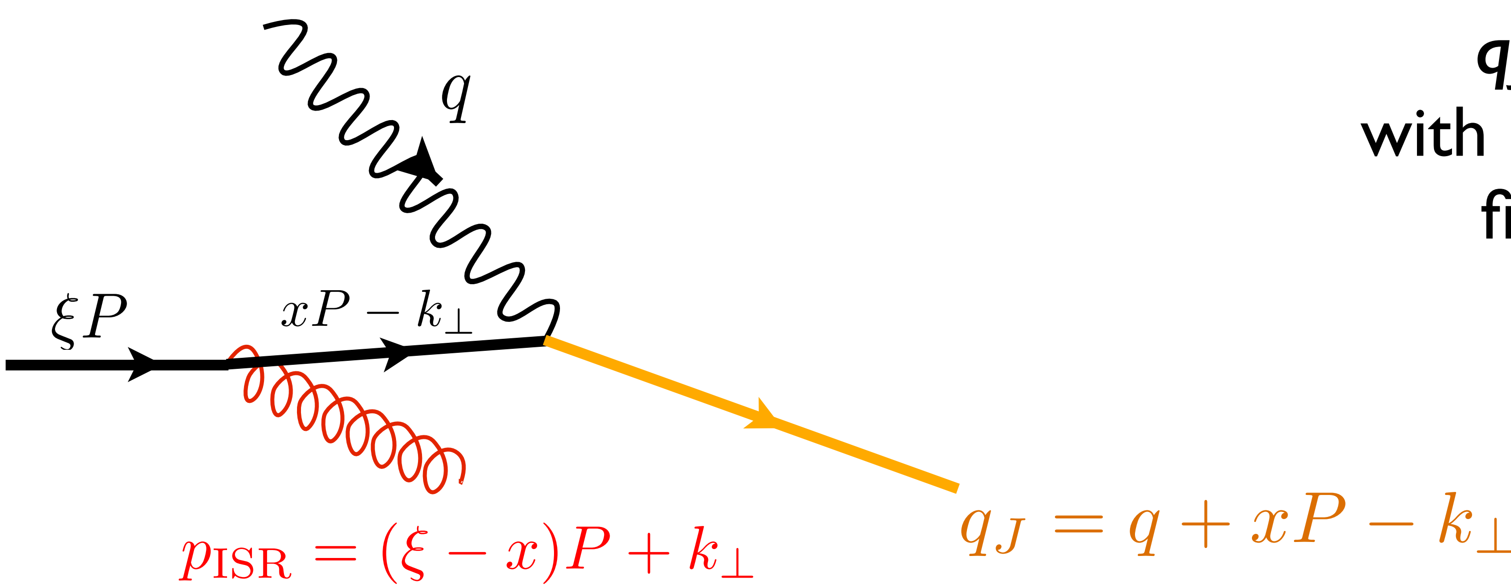
$$q_B = xP$$

$$q_J = \text{true jet axis}$$

[similar but not identical definition in Z. Kang, Mantry, Qiu (2012)]

q_J is **A**ligned with the jet momentum, with no relative label transverse momentum: find by jet algorithm or minimization

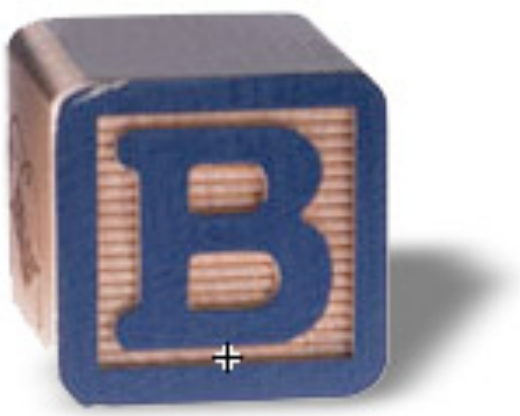
⇨ depends on momenta of final-state hadrons



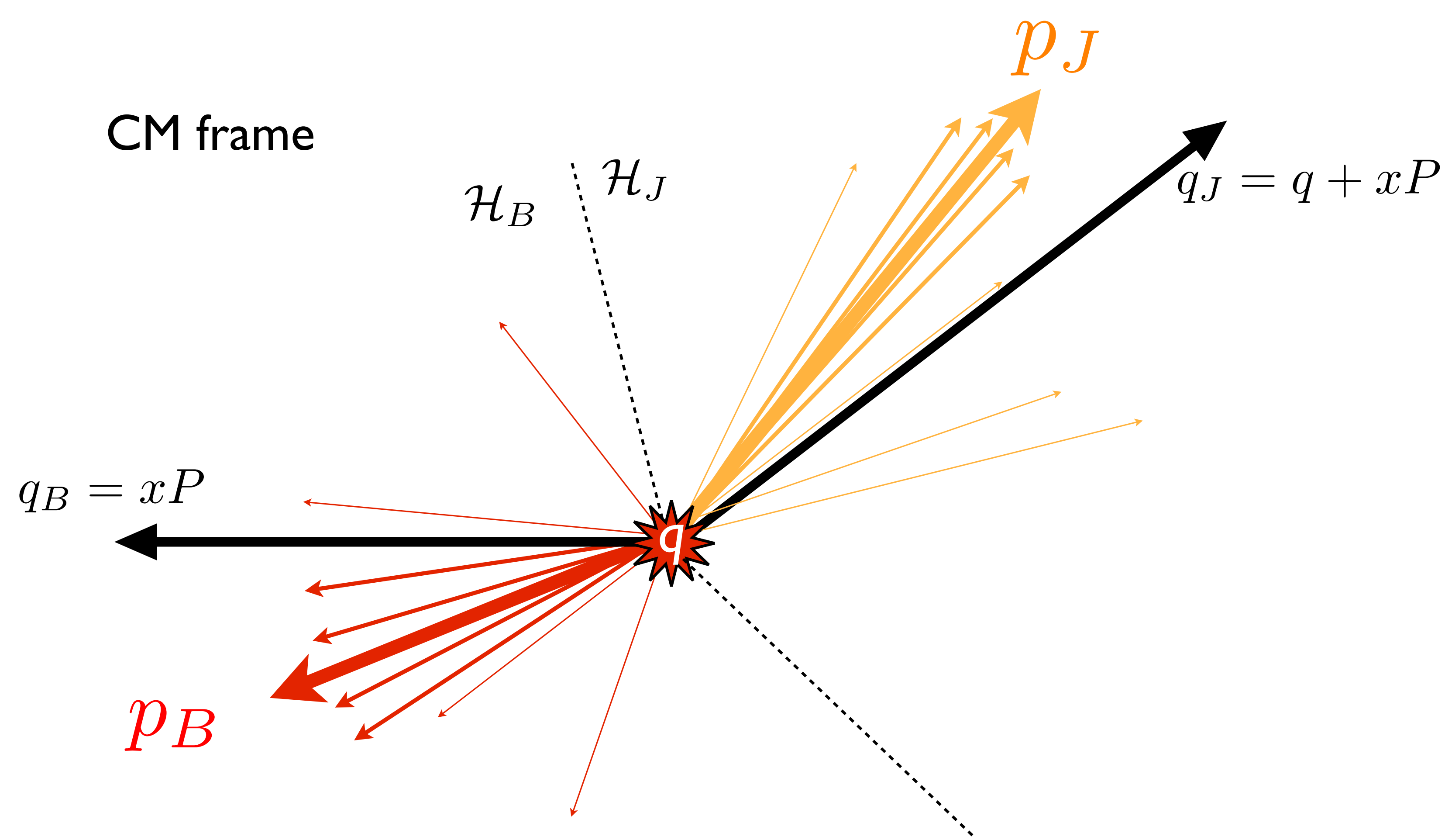
$k_\perp \sim Q\lambda$

Choices for DIS 1-jettiness

D. Kang, CL, Stewart (2013)



τ_1^b

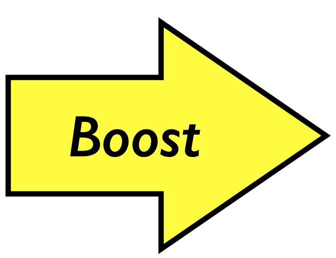


$$q_B = xP$$

$$q_J = q + xP$$

same as DIS thrust
by Antonelli, Dasgupta, Salam (1999)

***q_J* no longer exactly aligned with jet, but simpler in that *$q+xP$* is given only by lepton and initial-state proton momenta**

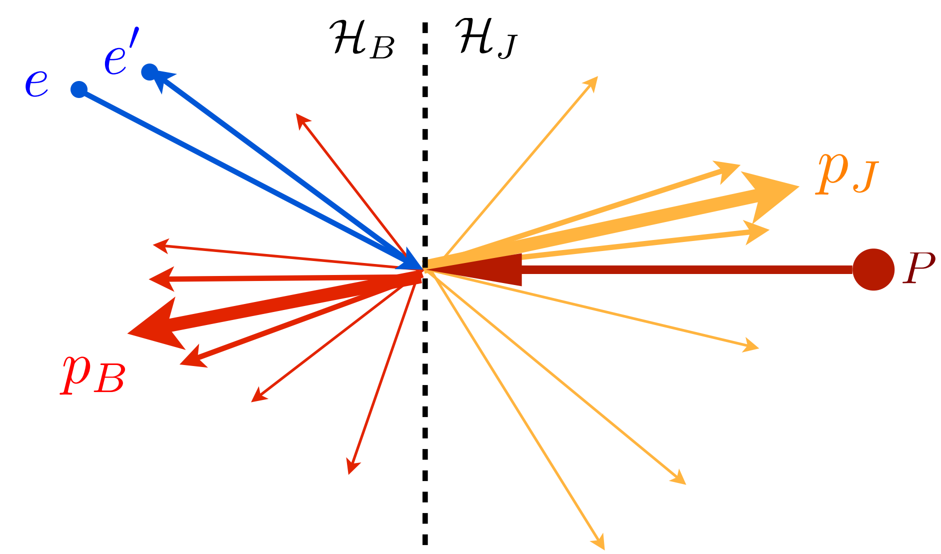


Breit frame:

$$q = (Q, 0, 0, Q)$$

$$q_B = Q\bar{n}_z \quad q_J = Qn_z$$

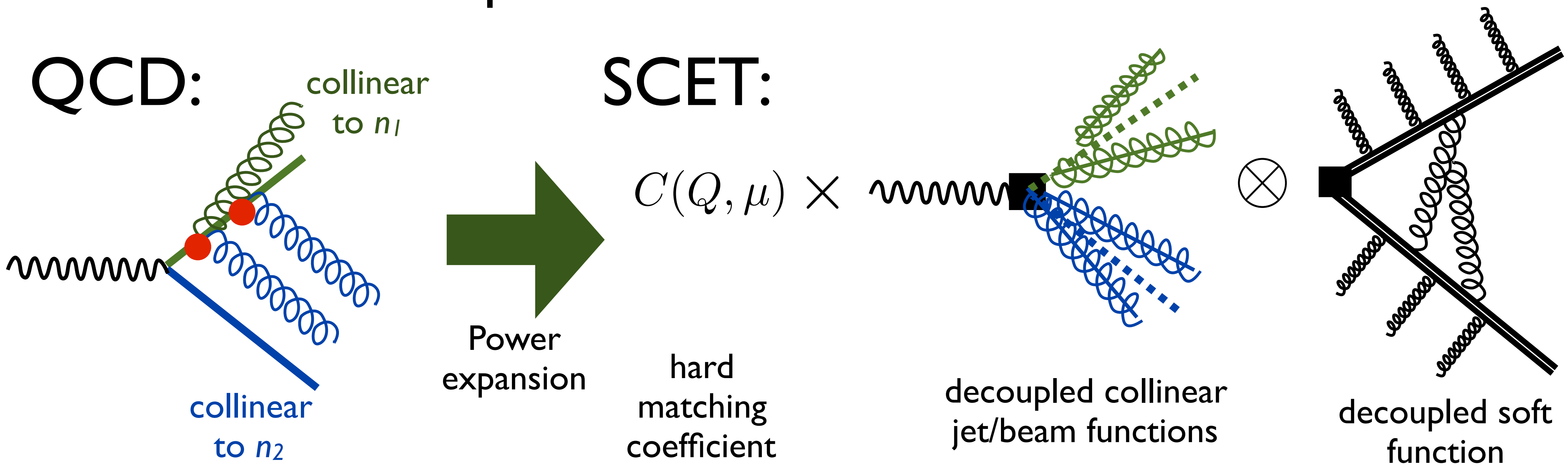
1-jettiness regions are hemispheres in Breit frame



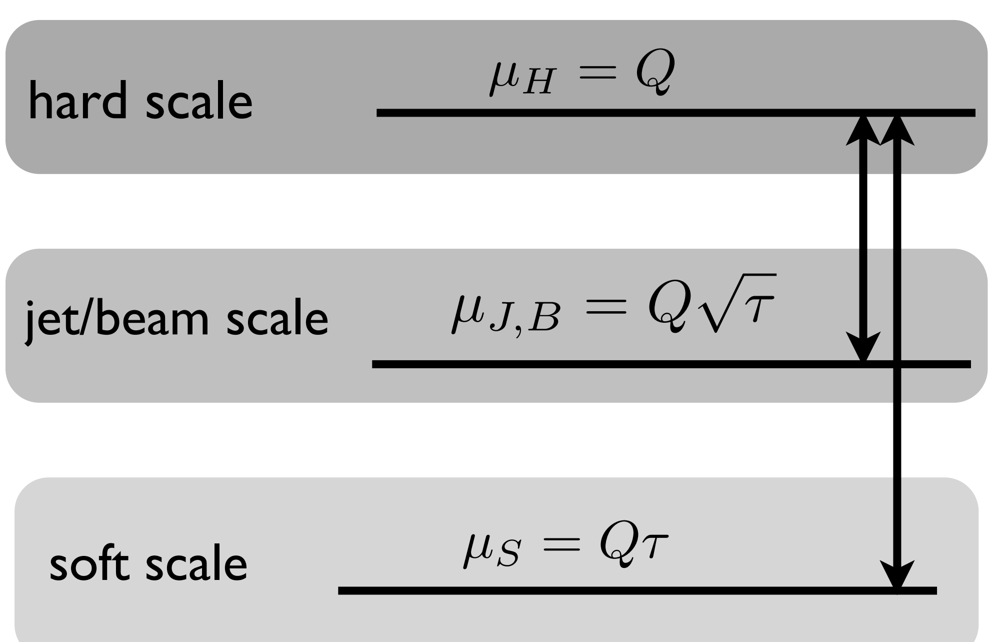
[See D. Kang, CL, Stewart (2013) for a third version of 1-jettiness]

Soft Collinear Effective Theory

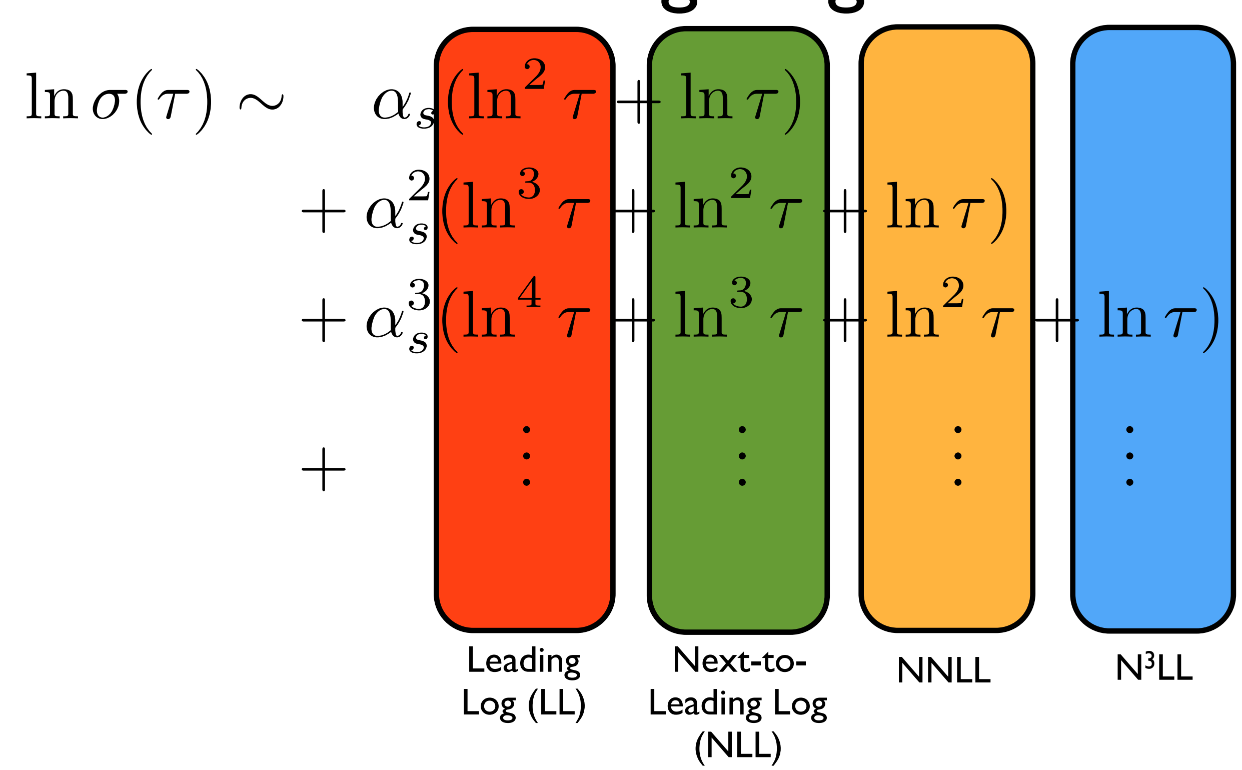
- Modern tools for high precision resummation, factorization of perturbative and nonperturbative effects Bauer, Fleming, Luke, Pirjol, Stewart (1999-2001)



- RG Evolution



- Resummation of large logs



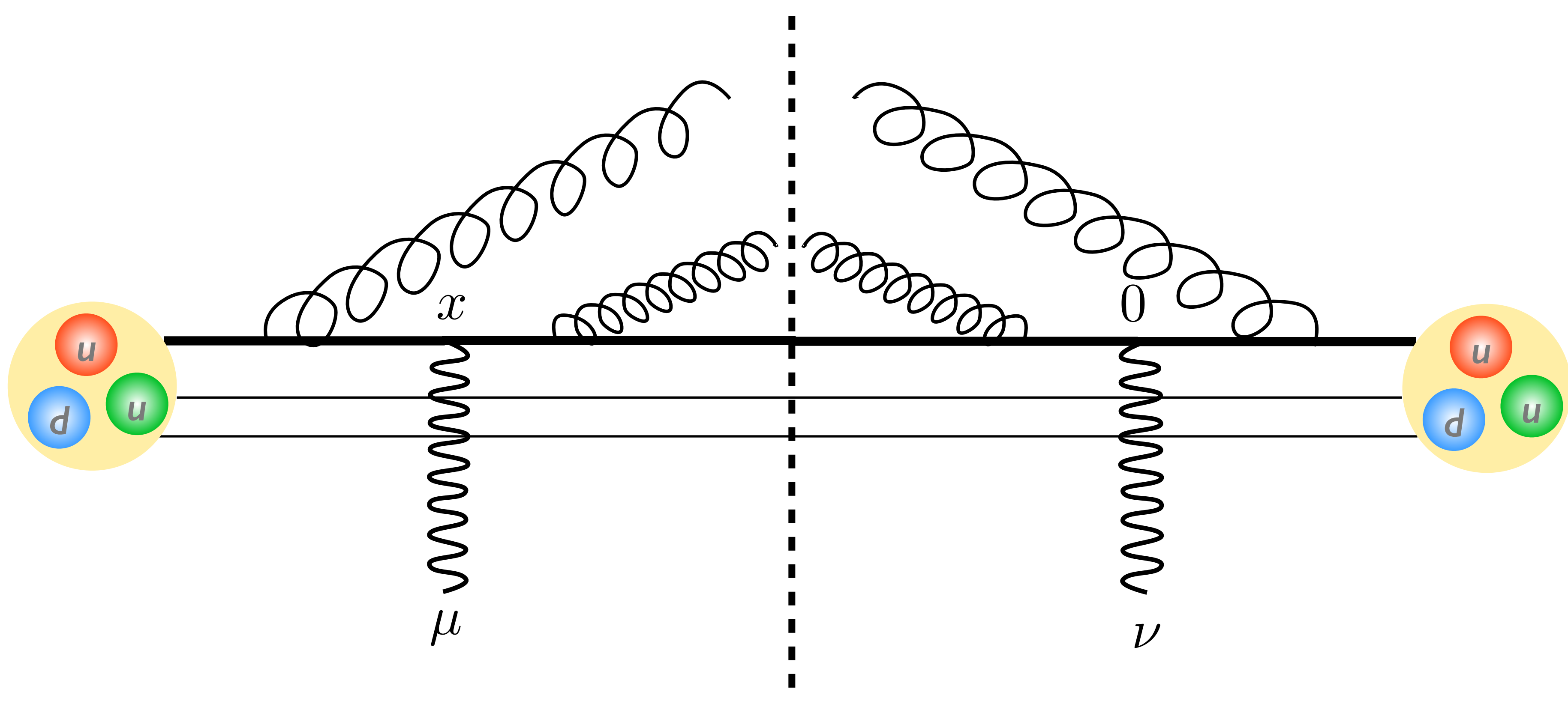
Factorization Theorem for 1-Jettiness

$$\frac{d\sigma(x,Q^2)}{d\tau_1} = \underbrace{L_{\mu\nu}(x,Q^2)}_{\text{leptonic tensor}}\underbrace{W^{\mu\nu}(x,Q^2,\tau_1)}_{\text{hadronic tensor}}$$

Start in QCD:

$$W^{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x}\langle P|\bar{q}\gamma^\mu q(x)\delta(\tau_1 - \hat{\tau}_1)\bar{q}\gamma^\nu q(0)|P\rangle$$

$$\hat{\tau}_1|X\rangle = \tau_1(X)|X\rangle$$



Measure τ_1 of particles crossing the cut

Factorization Theorem for 1-Jettiness

$$\frac{d\sigma(x,Q^2)}{d\tau_1} = L_{\mu\nu}(x,Q^2)W^{\mu\nu}(x,Q^2,\tau_1)$$

Match onto 2-jet operators in SCET:

$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^4x\, e^{iq\cdot x} \sum_{n_1,n_2} \int d^3\tilde{p}_1 d^3\tilde{p}_2 e^{i(\tilde{p}_2-\tilde{p}_1)\cdot x} C_\mu^*(\tilde{p}_1,\tilde{p}_2) C_\mu(\tilde{p}_1,\tilde{p}_2)$$

$$\times \langle P_{n_B}|\bar{\chi}_{n_2,\tilde{p}_2}(x)\overline{T}[Y_{n_2}^\dagger(x)Y_{n_1}(x)]\chi_{n_1,\tilde{p}_1}(x)$$

$$\times\, \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$

$$\times\, \bar{\chi}_{n_1,\tilde{p}_1}(0)T[Y_{n_1}^\dagger(0)Y_{n_2}(0)]\chi_{n_2,\tilde{p}_2}(0)|P_{n_B}\rangle$$

collinear jet operators in SCET

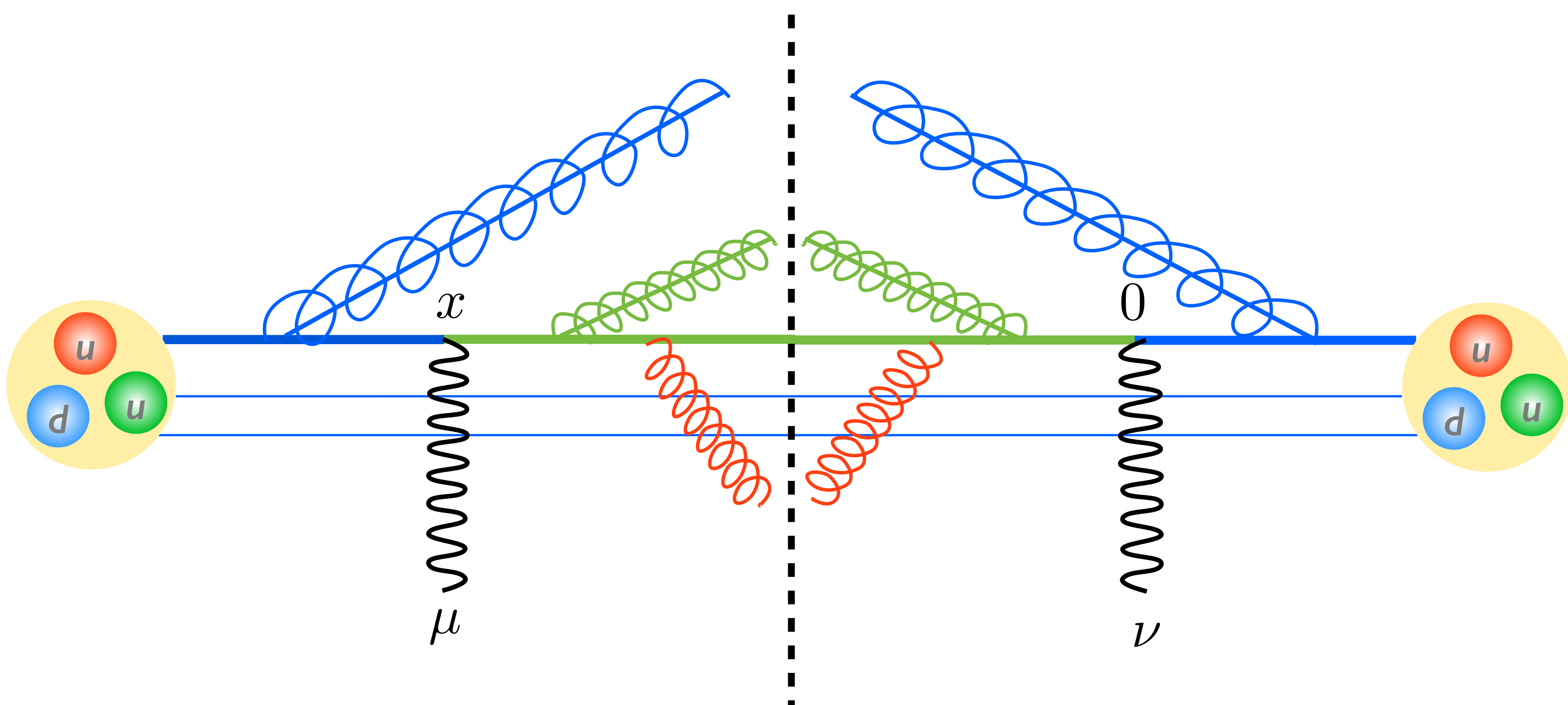
$$\chi_n = [W_n \xi_n]$$

collinear Wilson line

collinear quark field

soft gluon Wilson lines

$$Y_{n_{1,2}}$$



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$$\times \langle P_{n_B}|\bar{\chi}_{n_2,\tilde{p}_2}(x)\overline{T}[Y^\dagger_{n_2}(x)Y_{n_1}(x)]\chi_{n_1,\tilde{p}_1}(x)$$
$$\times\, \delta(\tau_1 - \hat{\tau}_1^{n_1} - \hat{\tau}_1^{n_2} - \tau_1^s)$$
$$\times\, \bar{\chi}_{n_1,\tilde{p}_1}(0) T[Y^\dagger_{n_1}(0)Y_{n_2}(0)]\chi_{n_2,\tilde{p}_2}(0)|P_{n_B}\rangle$$

collinear jet operators in SCET

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collinear Wilson line — collinear quark field

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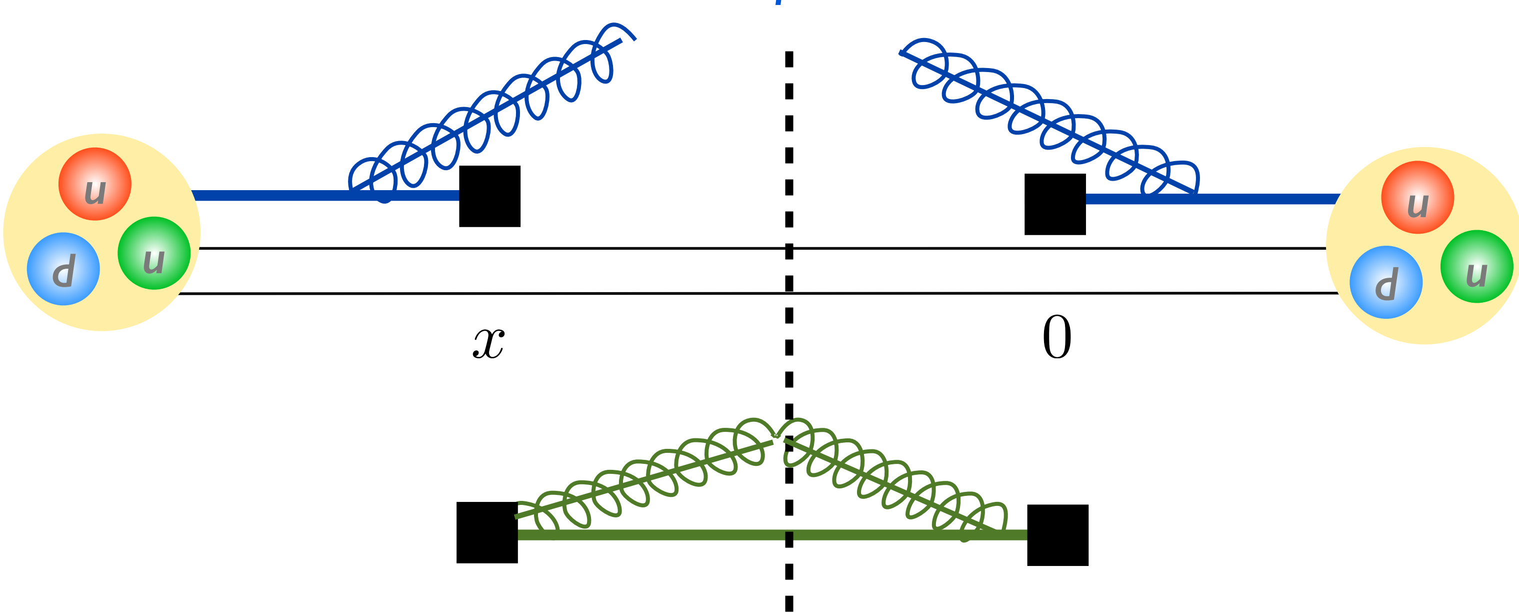
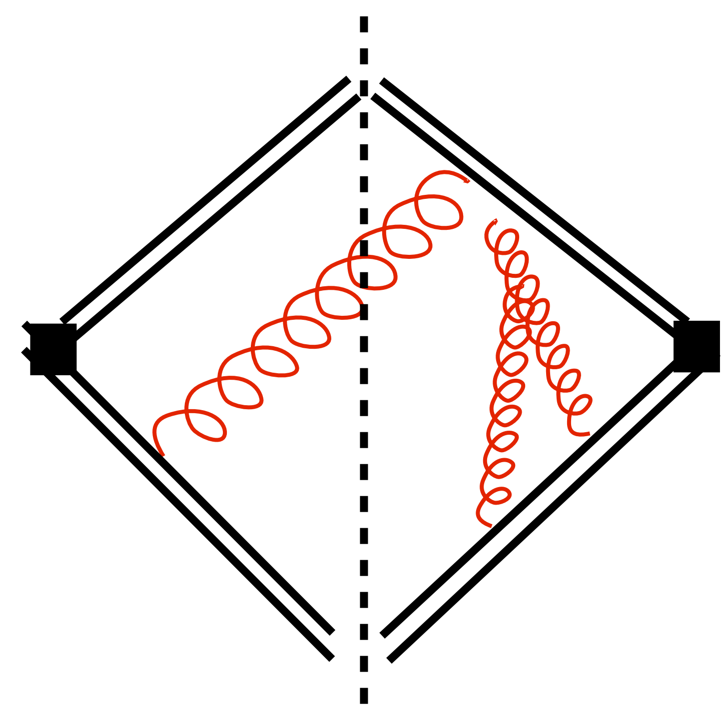
$$Y_{n_{1,2}}$$

"beam function"



"jet function"



"soft function"

Factorization Theorem for 1-Jettiness

Factor collinear and soft matrix elements:

hard function

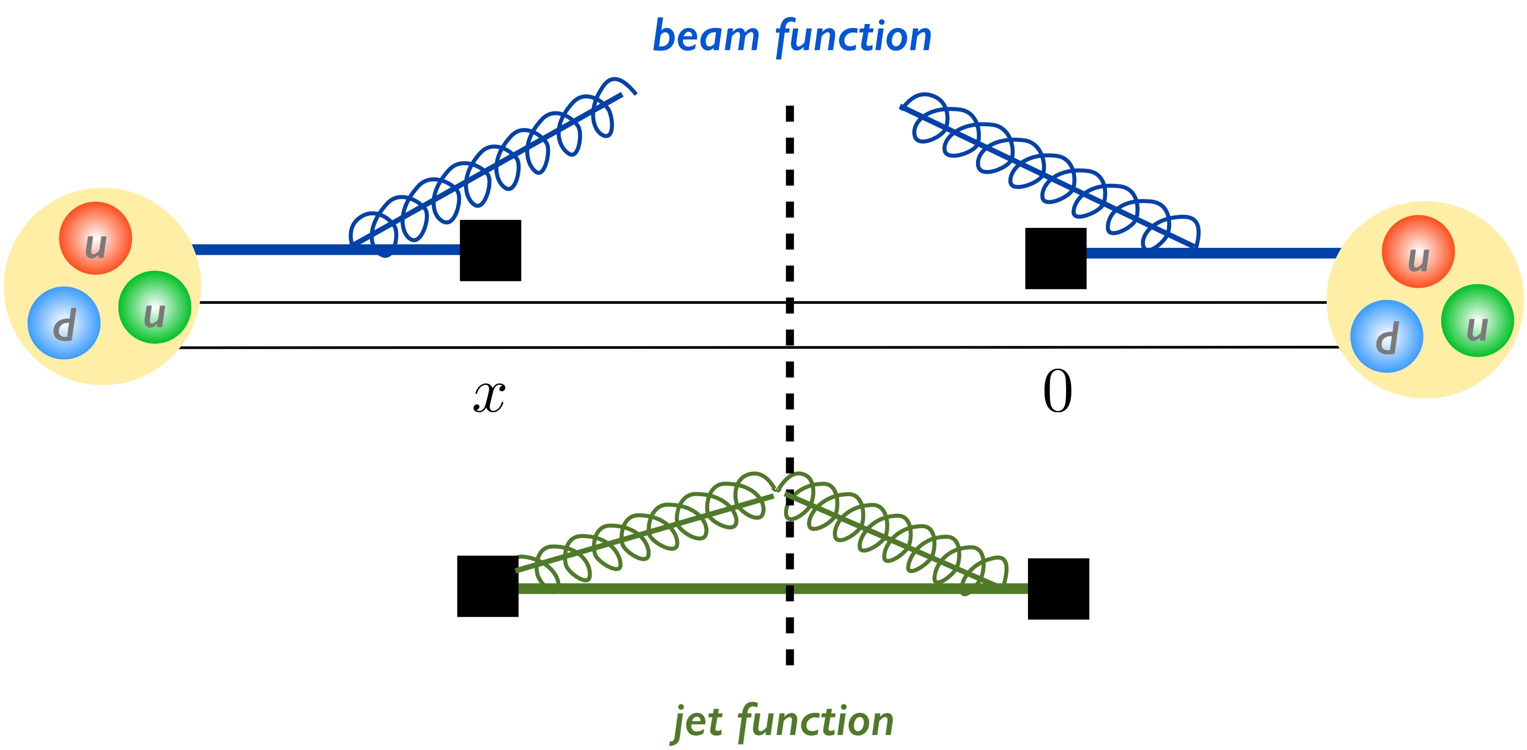
$$W_{\mu\nu}(x,Q^2,\tau_1) = \int d^2\tilde{p}_\perp \int d\tau_J d\tau_B d\tau_S \, C^*(Q^2,\mu)C(Q^2,\mu)\,\delta\Big(\tau_1 - \frac{t_J}{s_J} - \frac{t_B}{s_B} - \frac{k_S}{Q_R}\Big)$$

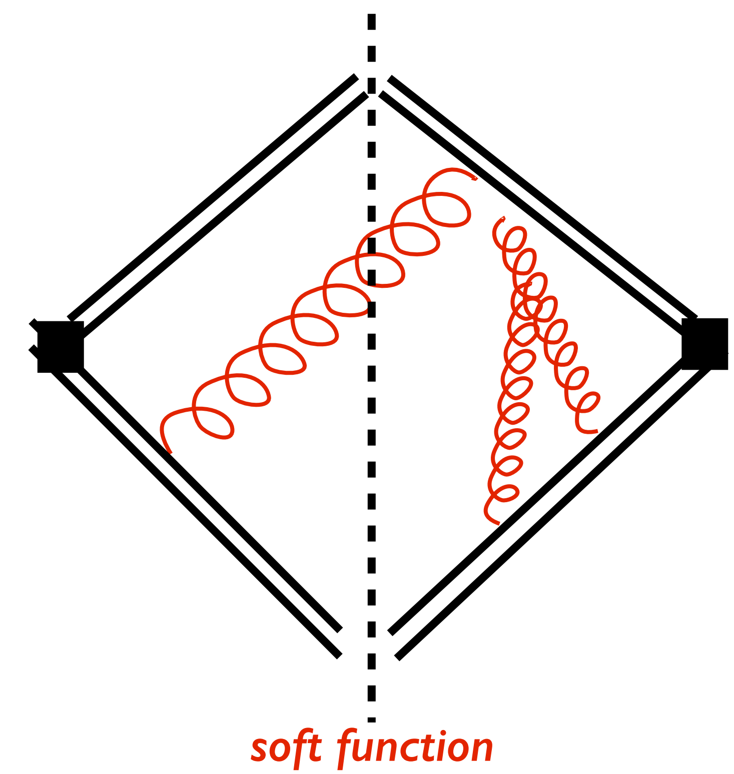
soft function $\times \langle 0|[Y^\dagger_{n'_J} Y^\dagger_{n'_B}](0)\delta(k_S - n'_J\cdot\hat{p}_{J'} - n'_B\cdot\hat{p}_{B'})[Y_{n'_B}Y_{n'_J}](0)|0\rangle$

beam function $\times \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)\chi_{n_B}](0)|P_{n_B}\rangle$

jet function $\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle$

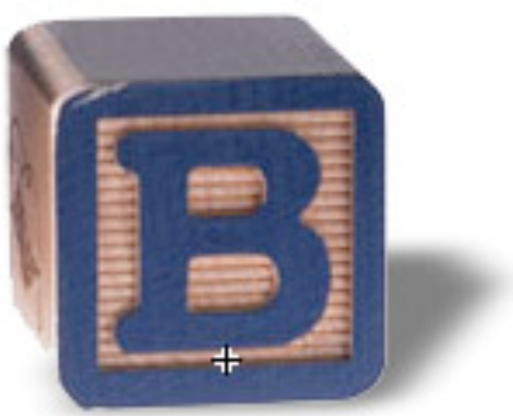
(+ permutations)





Factorization Theorems for 1-Jettiness

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\Big(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$



$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J - \mathbf{p}_\perp^2,\mu)\mathcal{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) S(k_S,\mu)$$

τ_1^a: jet momentum aligned with 1-jettiness axis, decoupled from beam p_T

τ_1^b: jet and beam p_T correlated by momentum conservation

➡ **difference in two distributions is a probe of ISR p_T**

Factorization Theorems for 1-Jettiness

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\Big(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
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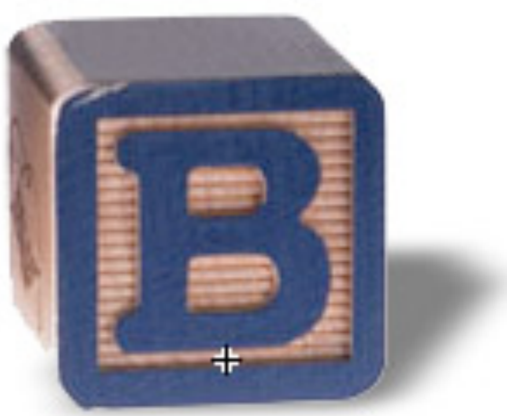

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^b} = H(Q^2,\mu)\int d^2p_\perp dt_J dt_B dk_S \delta\Big(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
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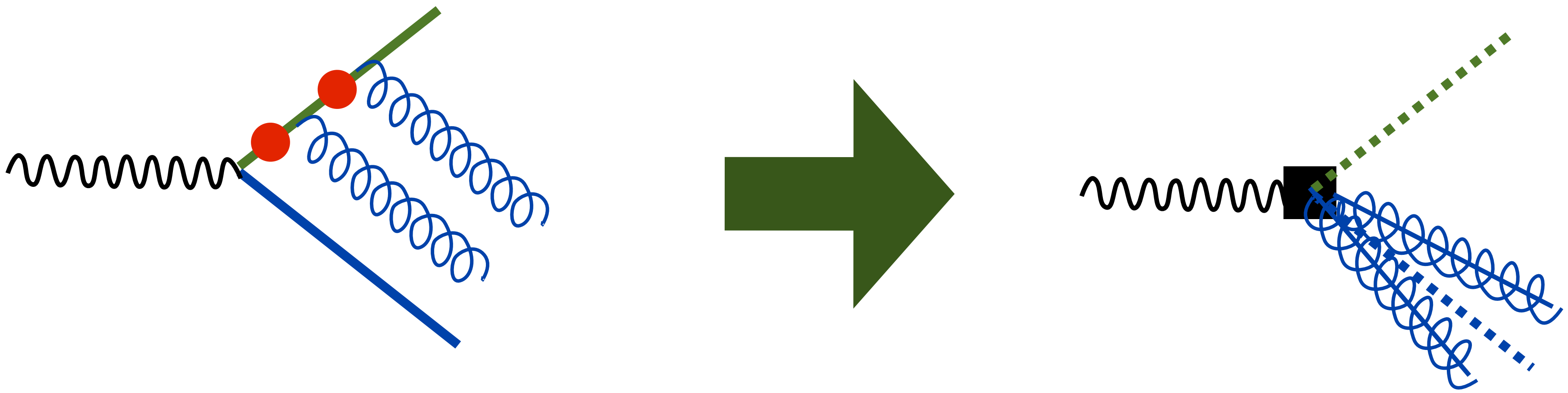
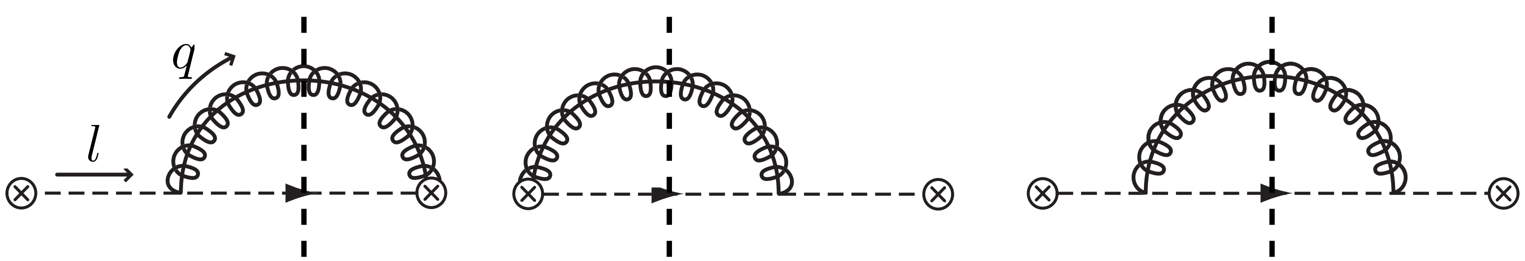
→ **difference in two distributions is a probe of ISR p_T**

Hard and Jet Functions



$$H(Q^2,\mu) = 1 + \frac{\alpha_s(\mu)C_F}{2\pi}\left(-\ln^2\frac{\mu^2}{Q^2} - 3\ln\frac{\mu^2}{Q^2} - 8 + \frac{\pi^2}{6}\right) + \ldots$$

known to 3 loops



$$J(t,\mu) = \delta(t) + \frac{\alpha_s(\mu)C_F}{4\pi}\left\{(7-\pi^2)\delta(t) - \frac{3}{\mu^2}\left[\frac{\mu^2\theta(t)}{t}\right]_+ + \frac{4}{\mu^2}\left[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\right]_+\right\} + \ldots$$

known to 2 loops

anomalous dimension known to 3 loops

Beam Function and PDFs

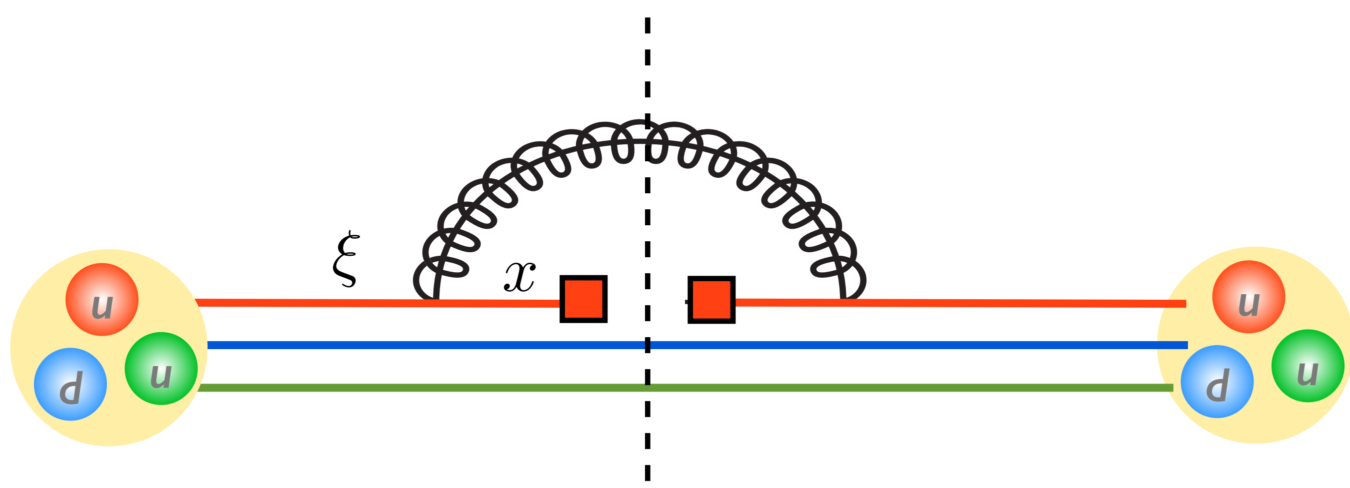
transverse momentum dependent beam function:

$$B(\omega k^+, x, k_\perp^2, \mu) = \frac{\theta(\omega)}{\omega} \int \frac{dy^-}{4\pi} e^{ik^+ y^-/2} \langle P_n(P^-)| \bar{\chi}_n\Big(y^- \frac{n}{2}\Big) \delta(xP^- - \bar{n}\cdot\mathcal{P}) \delta(k_\perp^2 - \mathcal{P}_\perp^2) \chi_n(0) | P_n(P^-)\rangle$$

match onto PDF

$$f(x, \mu) = \theta(\omega) \langle P_n(P^-)| \bar{\chi}_n(0) \delta(xP^- - \bar{n}\cdot\mathcal{P}) \chi_n(0) | P_n(P^-)\rangle$$

$$\mathcal{B}_q(t, x, \mathbf{k}_\perp^2, \mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t, \frac{x}{\xi}, \mathbf{k}_\perp^2, \mu\Big) f_j(\xi, \mu)$$



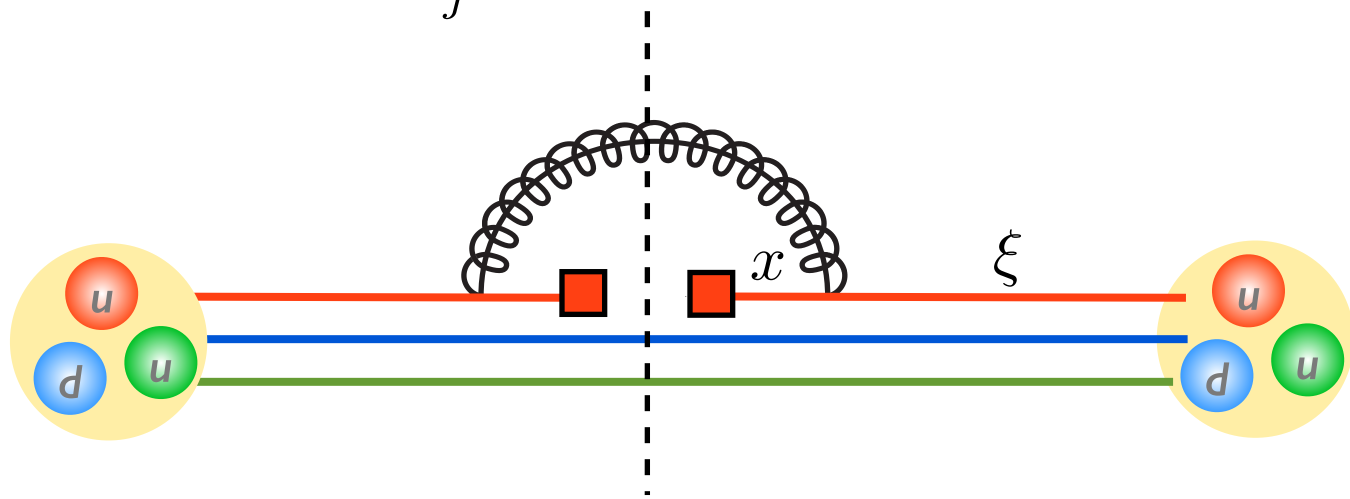
Measure small light-cone momentum $k^+ = t/P^-$
and transverse momentum $\mathbf{k}_\perp$
of initial state radiation

Generalized Beam Function to 1-loop

$$\mathcal{B}_q(t,x,\mathbf{k}_\perp^2,\mu) = \sum_j \int_x^1 \frac{d\xi}{\xi} \mathcal{I}_{ij}\Big(t,\frac{x}{\xi},\mathbf{k}_\perp^2,\mu\Big) f_j(x,\mu)$$

now known to 2 loops;
anomalous dimension
known to 3 loops

Gaunt, Stahlhofen, Tackmann (2014)



$$\mathcal{I}_{qq}(t,z,\mathbf{k}_\perp^2,\mu) = \frac{1}{\pi}\delta(t)\delta(1-z)\delta(\mathbf{k}_\perp^2) + \frac{\alpha_s(\mu)C_F}{2\pi^2}\theta(z)\Bigg\{\frac{2}{\mu^2}\bigg[\frac{\theta(t)\ln(t/\mu^2)}{t/\mu^2}\bigg]_+ \delta(1-z)\delta(\mathbf{k}_\perp^2)$$
$$+\frac{1}{\mu^2}\bigg[\frac{\theta(t)}{t/\mu^2}\bigg]_+ \bigg[P_{qq}(z) - \frac{3}{2}\delta(1-z)\bigg]\delta\Big(\mathbf{k}_\perp^2 - \frac{(1-z)t}{z}\Big)$$
$$+\delta(t)\delta(\mathbf{k}_\perp^2)\bigg[\bigg[\frac{\theta(1-z)\ln(1-z)}{1-z}\bigg]_+(1+z^2) - \frac{\pi^2}{6}\delta(1-z) + \theta(1-z)\Big(1-z-\frac{1+z^2}{1-z}\ln z\Big)\bigg]\Bigg\} \tag{162a}$$

Jain, Procura, Waalewijn (2009)

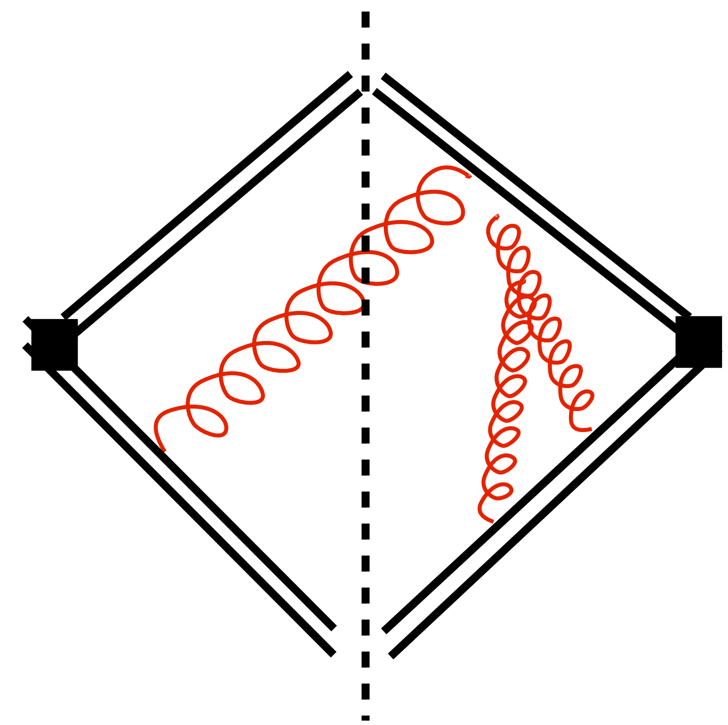
$$\mathcal{I}_{qg}(t,z,\mathbf{k}_\perp^2,\mu) = \frac{\alpha_s(\mu)T_F}{2\pi^2}\theta(z)\bigg\{\frac{1}{\mu^2}\bigg[\frac{\theta(t)}{t/\mu^2}\bigg]_+ P_{qg}(z)\delta\Big(\mathbf{k}_\perp^2 - \frac{(1-z)t}{z}\Big) + \delta(t)\delta(\mathbf{k}_\perp^2)\bigg[P_{qg}(z)\ln\frac{1-z}{z} + 2\theta(1-z)z(1-z)\bigg]\bigg\}, \tag{162b}$$

Tells us that PDFs should be evaluated at the beam radiation scale *t*

ordinary beam function: $B(t,x,\mu) = \int d^2k_\perp \mathcal{B}(t,x,\mathbf{k}_\perp^2,\mu)$

Stewart, Tackmann, Waalewijn (2009)

Soft Functions



- Definition of soft functions depends on direction of incoming/outgoing beams/jets:

outgoing jet: $$Y_n^{+\dagger}(x) = P\exp\left[ig\int_0^\infty ds\, n\cdot A_s(ns+x)\right]$$

incoming beam: $$Y_n^{-}(x) = P\exp\left[ig\int_{-\infty}^0 ds\, n\cdot A_s(ns+x)\right],$$

- Soft functions for e+e- dijets, DIS 1-jettiness, and *pp* beam thrust:

$$S_2(\ell_1,\ell_2,\mu) = \frac{1}{N_C}\mathrm{Tr}\sum_{i\in X_s}\left|\langle X_s|\,T[Y_n^{\pm\dagger}(0)Y_{\bar n}^{\pm}(0)]\,|0\rangle\right|^2$$
$$\times\,\delta\Big(\ell_1-\sum_{i\in X_s}\theta(\bar n\cdot k_i-n\cdot k_i)n\cdot k_i\Big)\delta\Big(\ell_2-\sum_{i\in X_s}\theta(n\cdot k_i-\bar n\cdot k_i)\bar n\cdot k_i\Big),$$

e$^+$e$^-$: $++$

DIS: $--$

***pp*:** $+-$

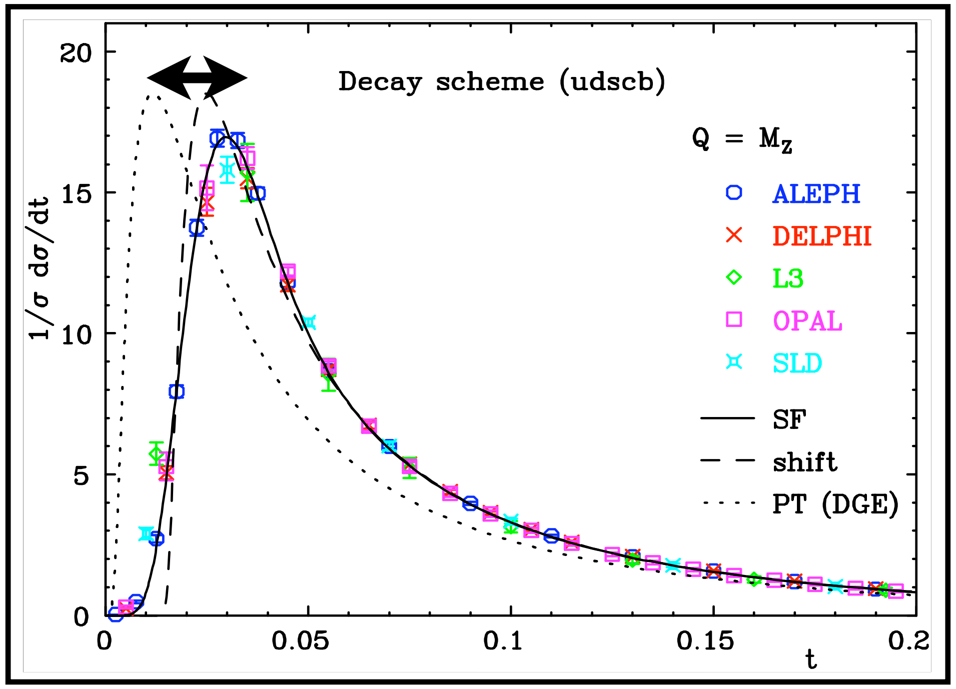
- Perturbatively, it is known that $\boxed{S_2^{ee}=S_2^{ep}=S_2^{pp}}$ to at least $\mathcal{O}(\alpha_s^2)$

Kang, Labun, CL (2015); Boughezal, Liu, Petriello (2015)

- Nonperturbatively, we cannot conclude anything about their equality, but…

NP Corrections

- Reminder: Dokshitzer-Webber model



$$\langle e \rangle = \langle e \rangle_{\rm PT} + c_e \frac{\Omega_1}{Q}$$

conjecture from single soft gluon emission: Dokshitzer, Webber (1995, 1997)

c_e observable dependent, calculable coefficient

Ω_1 universal nonperturbative parameter

(one for each of ee, *ep*, *pp*)

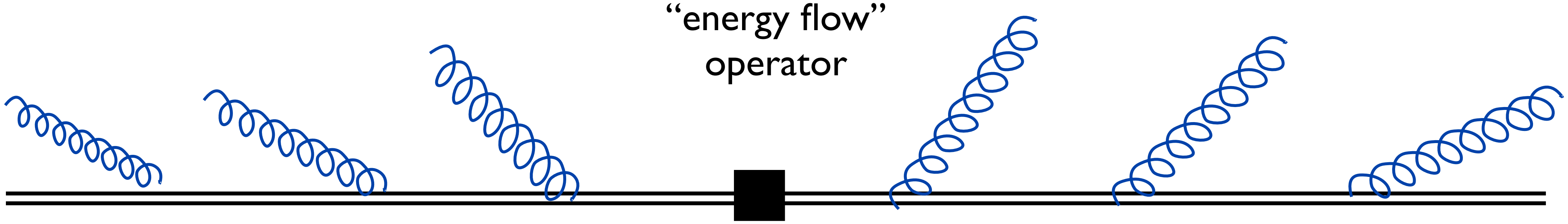
proof to all orders in soft gluon emission:

CL, Sterman (2006, 2007)

- SCET: First rigorous proof (and **field theory** definition of Ω_1) from factorization theorem and boost invariance of soft radiation:

$$\Omega_1 = \frac{1}{N_C} \mathrm{Tr} \langle 0 | \overline{Y}_{\bar n}^\dagger Y_n^\dagger \mathcal{E}_T(\eta) Y_n \overline{Y}_{\bar n} | 0 \rangle$$

"energy flow" operator



soft radiation sees only direction, not energy, of original collinear partons, invariant to boosts along z

Belitsky, Korchemsky, Sterman (2001)
Bauer, Fleming, **CL**, Sterman (2008)

Momentum Flow Operators

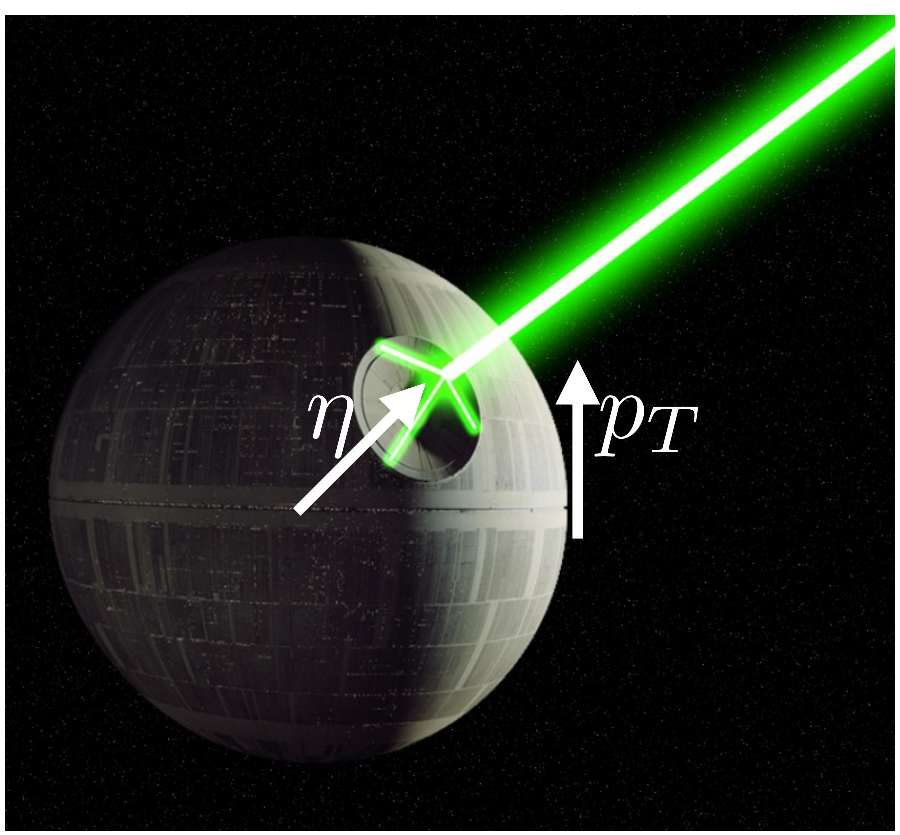
generic form of event shapes: $e(X) = \frac{1}{Q}\sum_{i\in X} f_e(\eta_i)|\mathbf{p}_T^i|$ e.g. angularities $f_{\tau_a}(\eta) = e^{-|\eta|(1-a)}$

operator action in terms of transverse momentum flow operator:

$$\hat{e}\,|X\rangle \equiv e(X)\,|X\rangle = \frac{1}{Q}\int_{-\infty}^{\infty} d\eta\, f_e(\eta)\mathcal{E}_T(\eta;\hat{t})\,|X\rangle$$

$$\mathcal{E}_T(\eta)|X\rangle = \sum_{i\in X}|\mathbf{p}_T^i|\delta(\eta-\eta_i)|X\rangle$$

construct out of energy-momentum tensor of QCD:



$$\mathcal{E}_T(\eta) = \frac{1}{\cosh^3\eta}\int_0^{2\pi} d\phi \lim_{R\to\infty} R^2 \int_0^\infty dt\,\hat{n}_i T_{0i}(t,R\hat{n})$$

measures total transverse momentum $|\mathbf{p}_T|$ flowing through slice of sphere at rapidity η from collision time *t=0* to detector at $t\to\infty$

$R\to\infty$

since Lagrangian of SCET factors into collinear and soft sectors, so does the energy-momentum tensor:

$$T_{\mu\nu} \to T_{\mu\nu}^n + T_{\mu\nu}^{\bar{n}} + T_{\mu\nu}^s$$

CL, Sterman (2006, 2007)

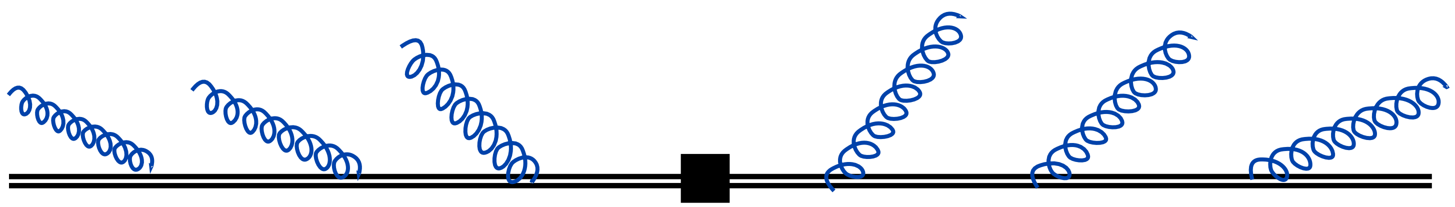
Proof of universality

- In general NP part of soft function must be modeled and is observable-dependent:

$$S(e,\mu,\Lambda) = \int_0^\infty de'\, S_{\rm PT}(e-e',\mu) F_{\rm NP}(e',\Lambda)$$

- The universality of the first moment, however, can be proven exactly:

$$\Delta\langle e\rangle_s = \frac{1}{Q}\int_{-\infty}^{\infty} d\eta\, f_e(\eta) \frac{1}{N_C}\,\mathrm{Tr}\langle 0|\overline{T}[Y_n^\dagger Y_{\bar n}]\mathcal{E}_T(\eta) T[Y_{\bar n}^\dagger Y_n]|0\rangle$$



$\Lambda_\alpha^{-1}\Lambda_\alpha$ Lorentz boosts by rapidity α along z:

$$Y_n = P\exp\left[ig\int_0^\infty ds\, n\cdot A_s(ns)\right] \Rightarrow Y_n$$

$$|0\rangle \Rightarrow |0\rangle$$

$$\mathcal{E}_T(\eta) \Rightarrow \mathcal{E}_T(\eta+\alpha)$$

$$\Rightarrow \boxed{\Delta\langle e\rangle_s = \frac{1}{Q}\underbrace{\left\{\int_{-\infty}^{\infty} d\eta\, f_e(\eta)\right\}}_{c_e}\underbrace{\left\{\frac{1}{N_C}\,\mathrm{Tr}\langle 0|\overline{T}[Y_n^\dagger Y_{\bar n}]\mathcal{E}_T(0) T[Y_{\bar n}^\dagger Y_n]|0\rangle\right\}}_{\Omega_1}}$$

e.g. $c_\tau = 2$ $\quad c_C = 3\pi$ $\quad c_{\tau_a} = \dfrac{2}{1-a}$ for e+e- scaling is obeyed well by LEP data

Nonperturbative Soft Model Function

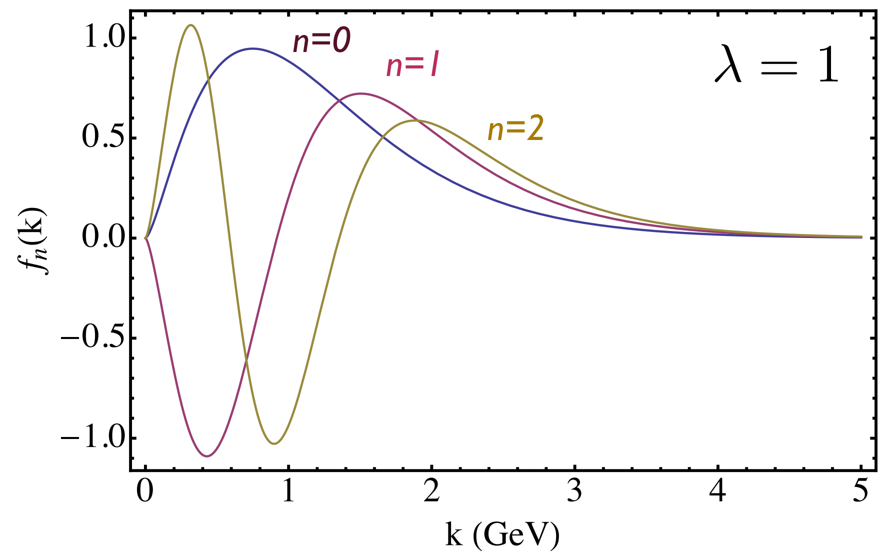
Convolution of perturbative soft function (soft radiation) with nonperturbative model function (hadronization):

$$S(k_S, \mu) = \int dl\, S_{\rm PT}(k_S - l, \mu) S_{\rm NP}(l)$$

$$S_{NP}(l) = f(l - \Delta)$$

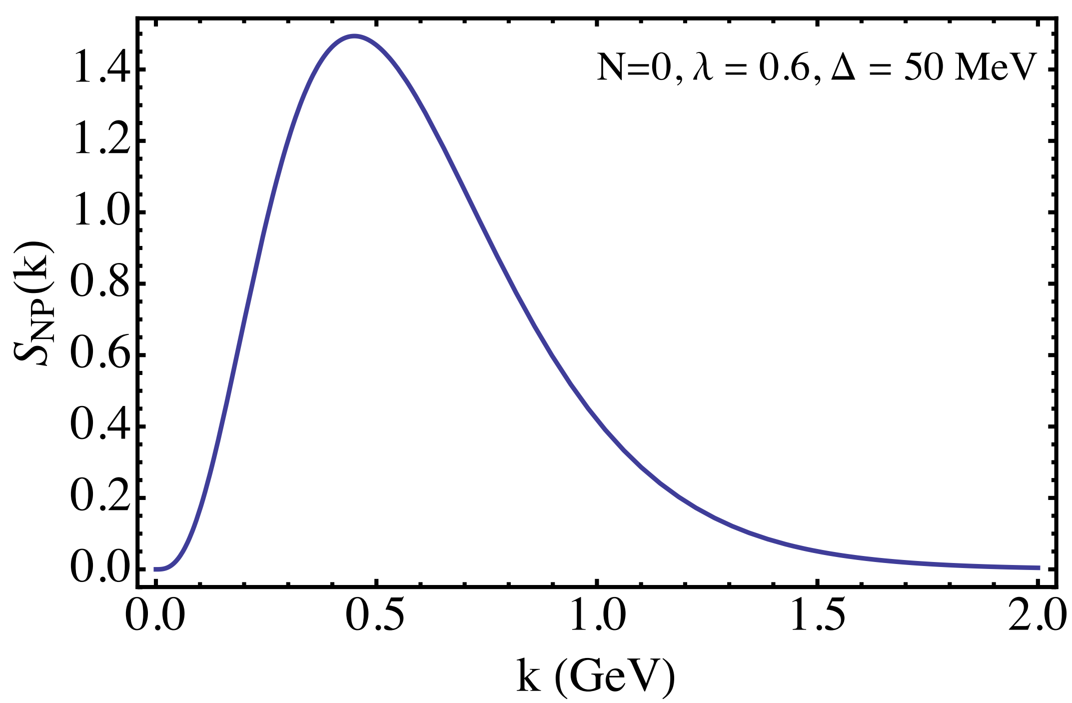
$$f(l) = \frac{1}{\lambda} \sum_{n=0}^{N} c_n f_n\Big(\frac{l}{\lambda}\Big)$$

Ligeti, Stewart, Tackmann (2008)



Basis coefficients, width and gap should be fit to data for one event shape and value of *Q*. Universality allows predictions for other event shapes and values of *Q*.

In following results, the following model function will be used:



Resummation of Logs

- Solution of RG Equations resums logs to all orders in α_s
- Order of logarithmic accuracy (LL, NLL, etc.) depends on accuracy to which anomalous dimensions and fixed-order matrix elements are known:

	Γ_F	γ_F	c_F	$\beta[\alpha_s]$
LL	α_s	1	1	α_s
NLL	α_s^2	α_s	1	α_s^2
NNLL	α_s^3	α_s^2	α_s	α_s^3
N³LL	α_s^4	α_s^3	α_s^2	α_s^4

previous accuracy for DIS thrust (1999)

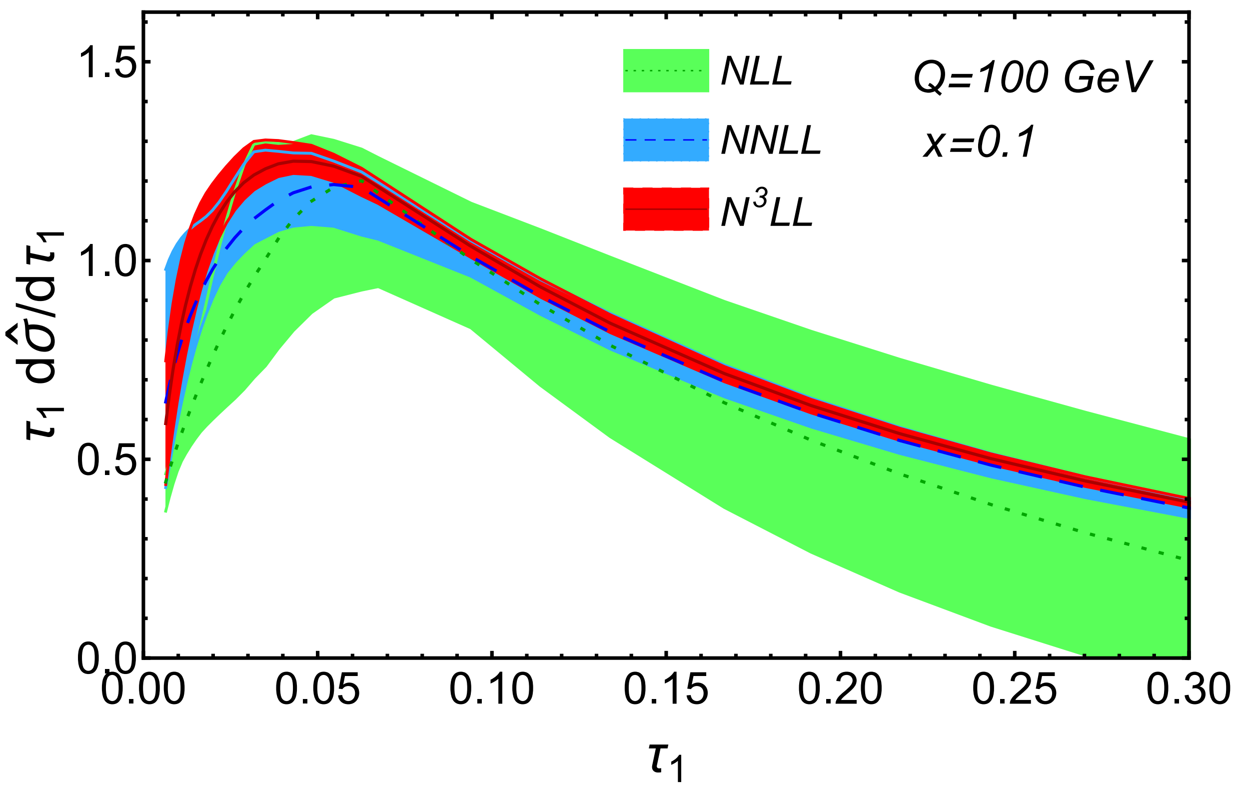
All* pieces now known for DIS 1-jettiness

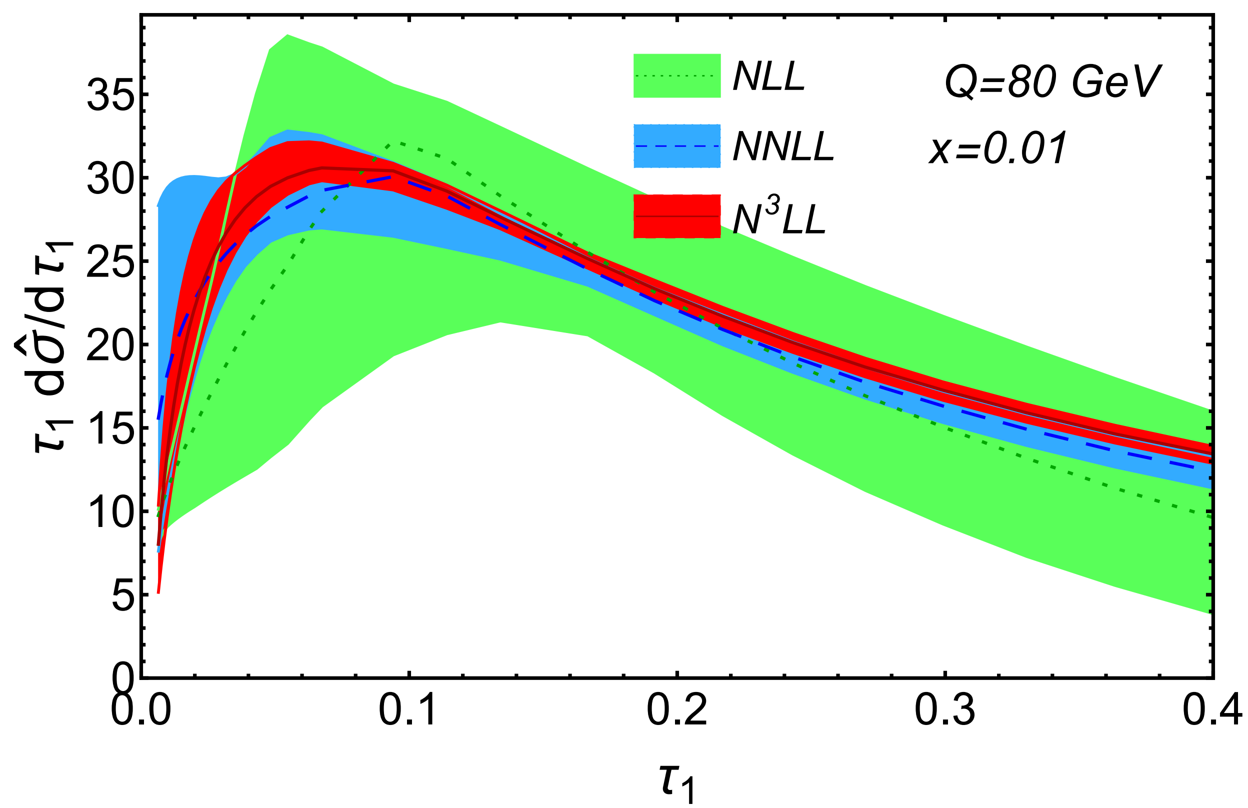
* cusp anom. dim. only to 3 loops, but unknown piece introduces small uncertainty

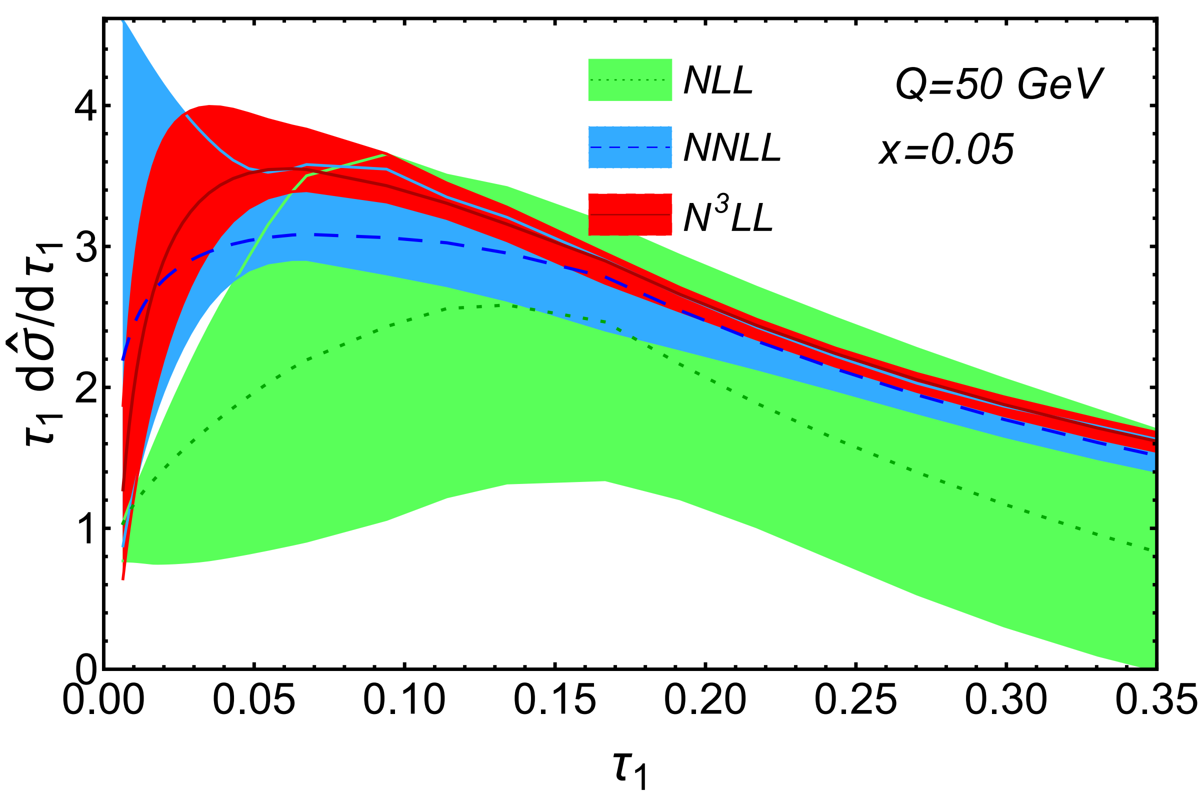
Predictions for DIS 1-jettiness

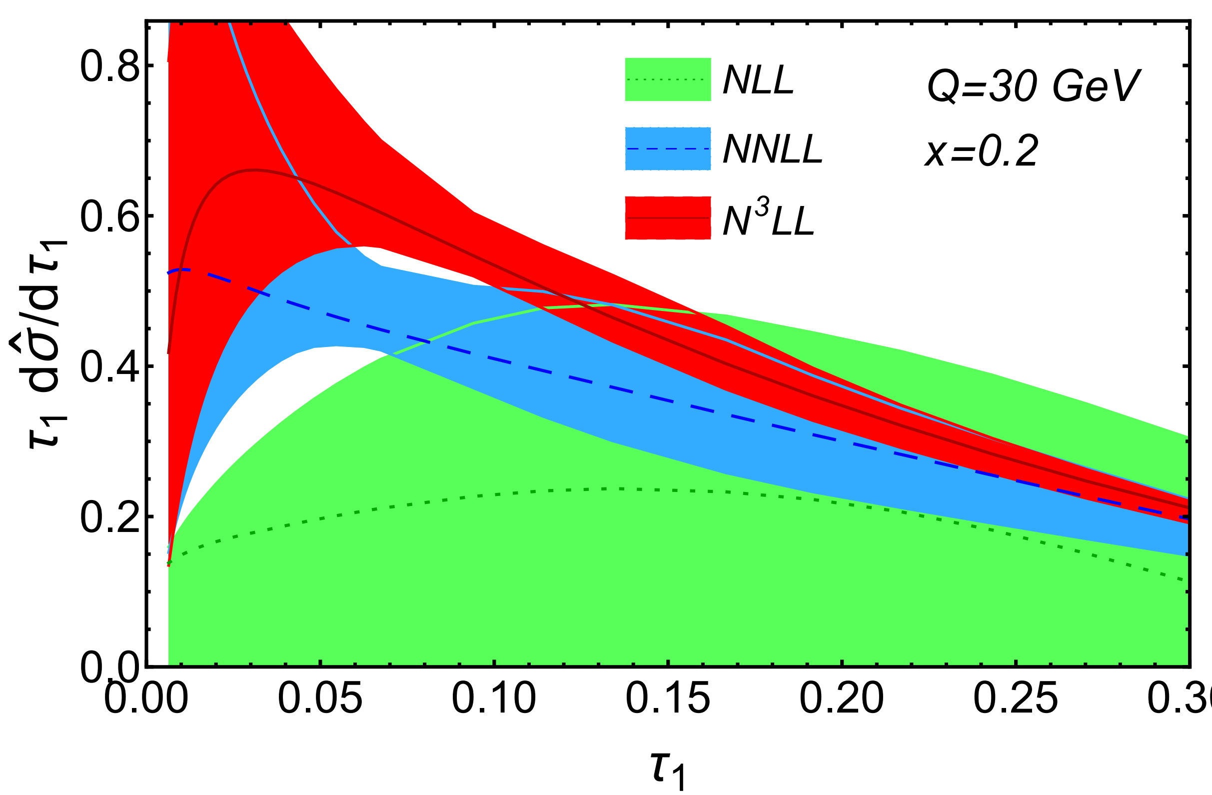
Partonic only, no hadronization, MSTW 2008 PDFs, matched to NLO fixed order (Kang, CL, Stewart 2014):

Preliminary: Kang, CL, Stewart (2016)



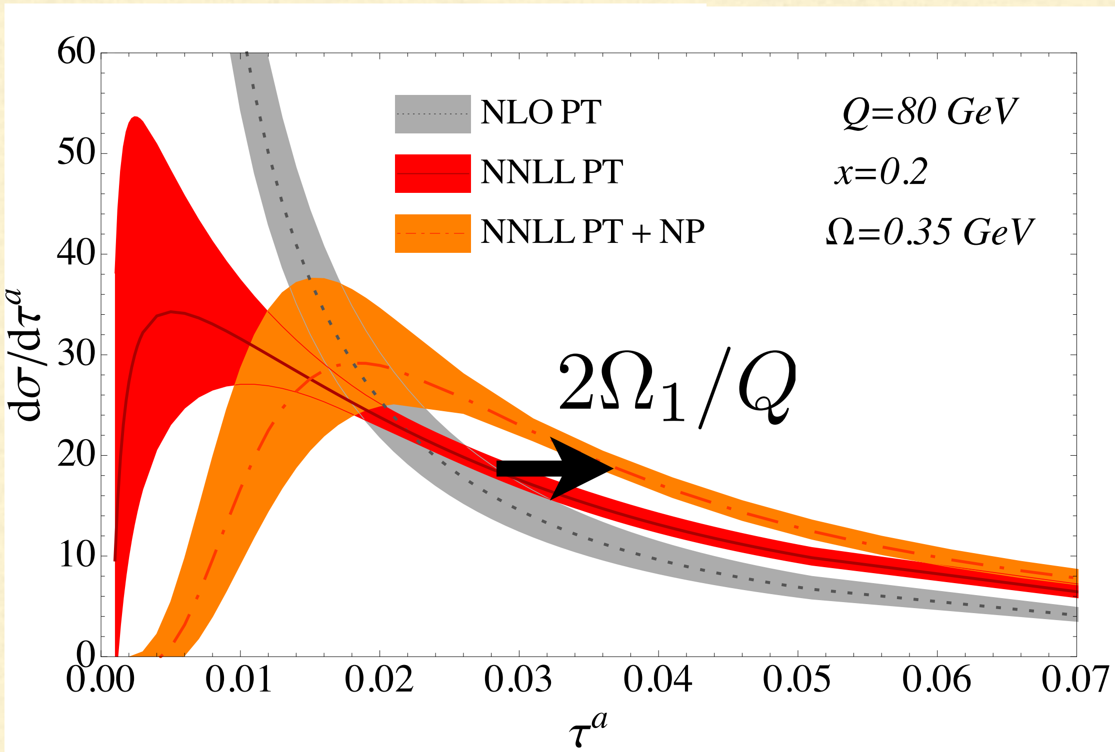






POWER CORRECTIONS IN *PP* AND DIS

- Universal nonperturbative shift in 3 versions of DIS 1-jettiness:

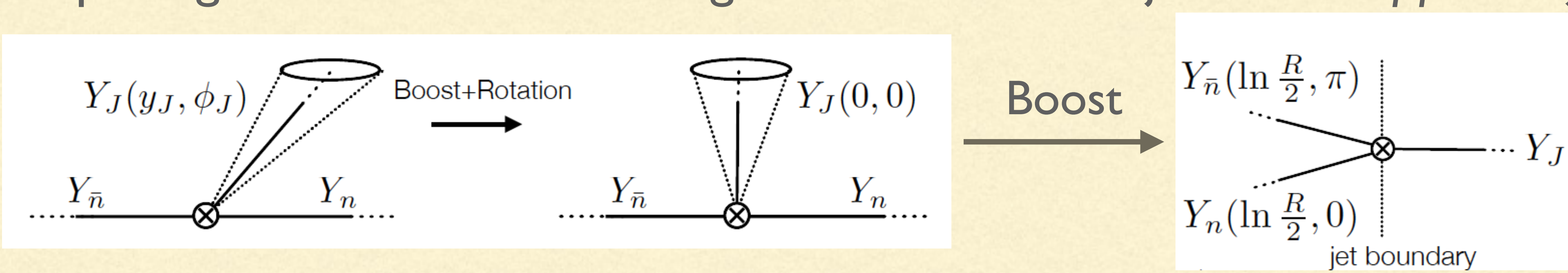


Using factorization theorems and boost invariance properties of soft Wilson lines, can prove that:

$$\boxed{\Omega_1^{\mathrm{a}} = \Omega_1^{\mathrm{b}} = \Omega_1^{\mathrm{c}}}$$

D. Kang, CL, I. Stewart (2013)

- Surprising relation also to leading NP correction to jet mass in *pp* to 1 jet



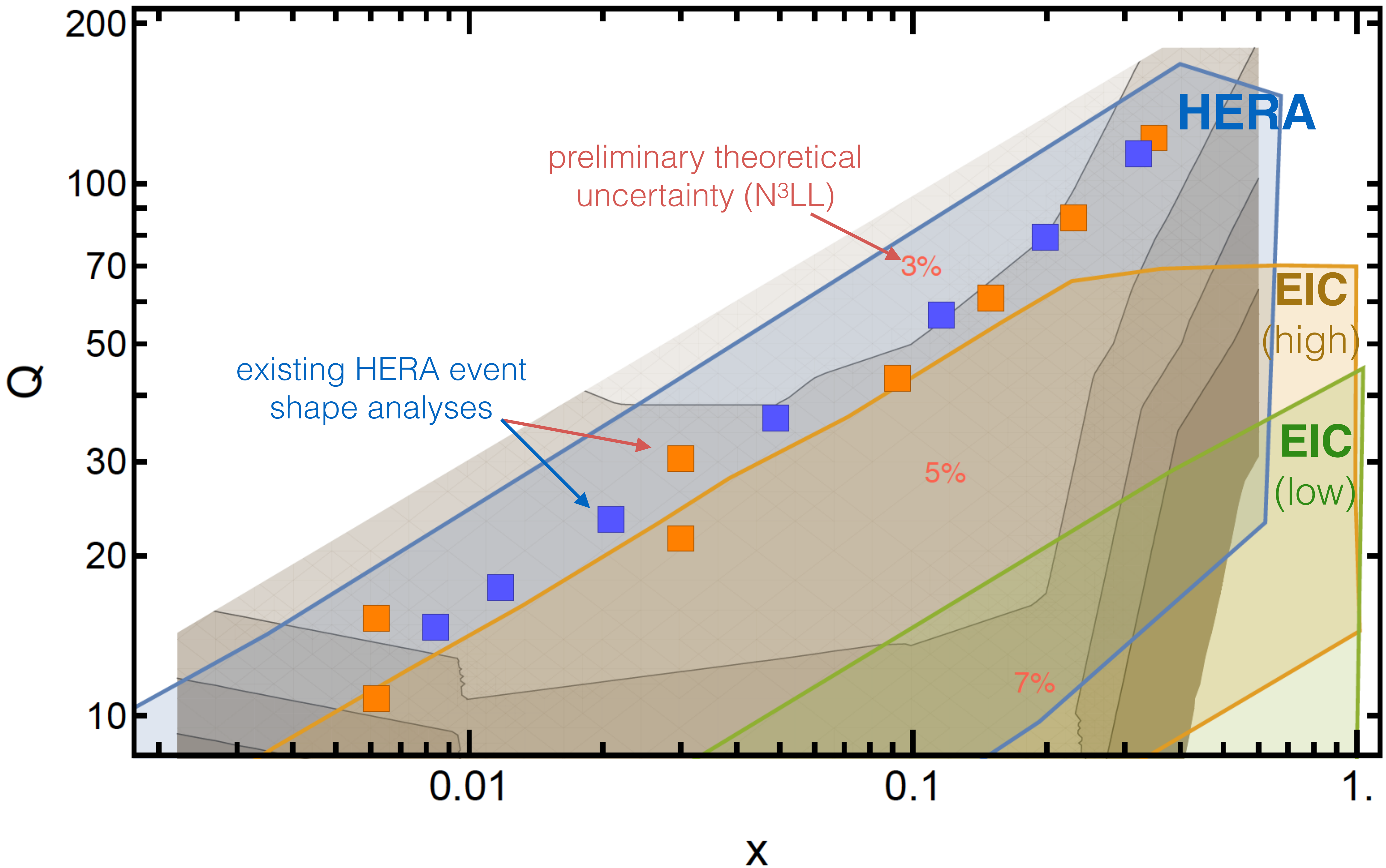
For $R \ll 1$, the beam Wilson lines fuse and $\Omega = \frac{R}{2}\,\Omega_0 + \ldots$

The universal Ω_0^q can be extracted from DIS event shapes

Stewart, Tackmann, Waalewijn (2014)

Experimental Coverage

Preliminary: Kang, CL, Stewart (2016)



New analyses of HERA data for 1-jettiness under way!

Some preliminary observations

- Few percent precision now achievable for 1-jettiness at relatively large Q and x
  - could achieve even better precision in determining strong coupling by fitting to many Q, x data sets.
- Data across many Q and x values also provide powerful test of universality of nonperturbative shift
- Lower Q data may prove useful to measure higher moments of nonperturbative shape functions
- Other observables? Energy-energy correlation (EEC) motivated suite of jet structure/substructures observables now available, amenable to similarly high-precision computation

e.g. Larkoski, Moult, Neill (2014-15);
Moult, Necib, Thaler (2016)

Summary

- We have computed 1-jettiness cross sections in DIS to N^{3}LL resummed accuracy
    - different versions of 1-jettiness probe ISR p_T differently, offer a test of universality of nonpeturbative effects
- SCET provided the tools:
    - to vastly improve the perturbative accuracy of resummed predictions
    - to identify universal nonperturbative effects on 1-jettiness distributions
- Jets in electron-proton collisions are a powerful probe of the strong coupling and hadron structure, with the requisite theoretical work and with the right machines.

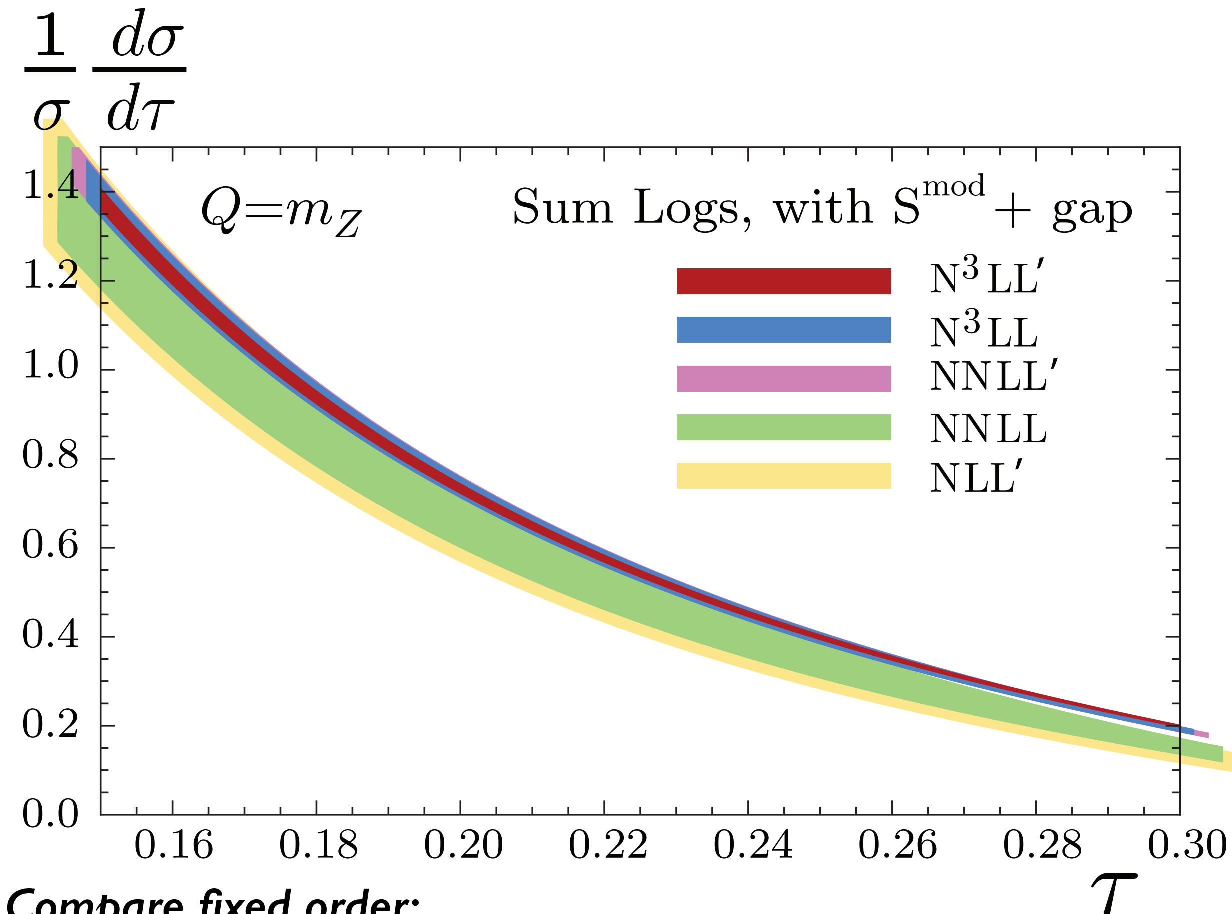
e^+e^- Thrust: Precision extraction of α_s

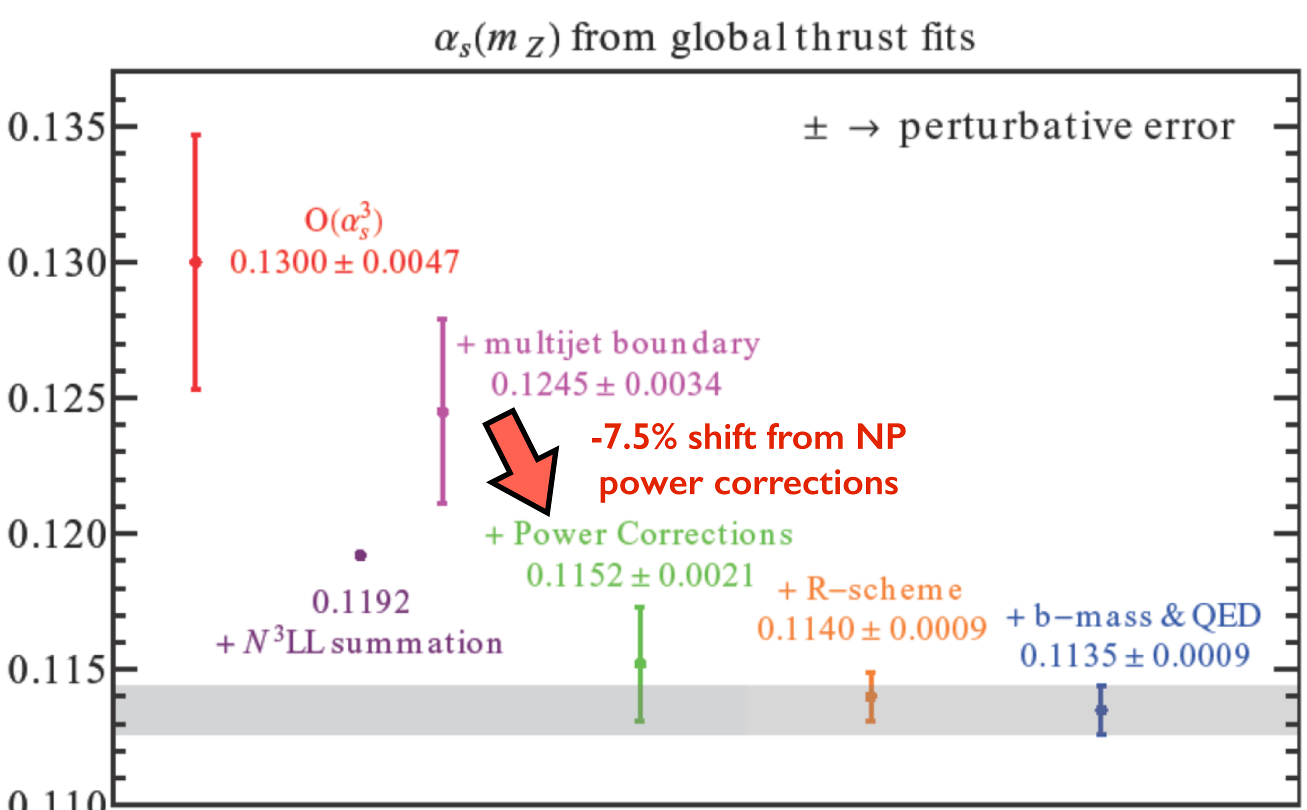
(2-jettiness)

NNNLL perturbative prediction + nonperturbative soft power correction led to most precise extraction of strong coupling from event shapes

Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

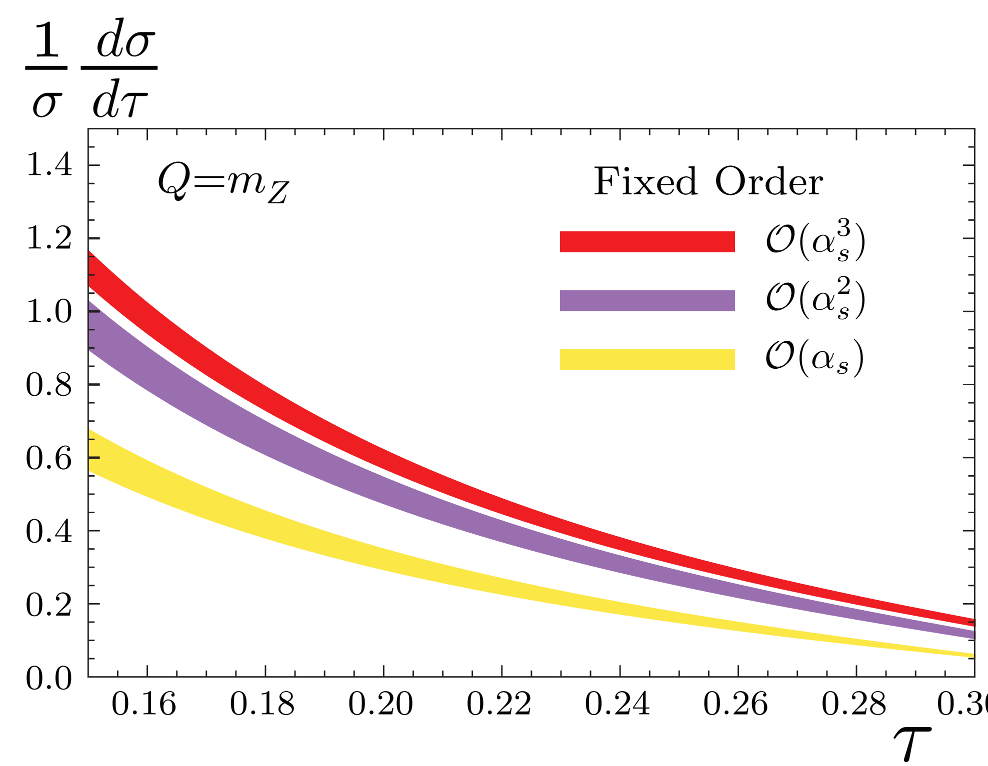
NNNLL resummed perturbative distribution — Becher, Schwartz (2008)





Abbate, Fickinger, Hoang, Mateu, Stewart (2010)

Compare fixed order:



e^+e^- Thrust: Precision extraction of α_s

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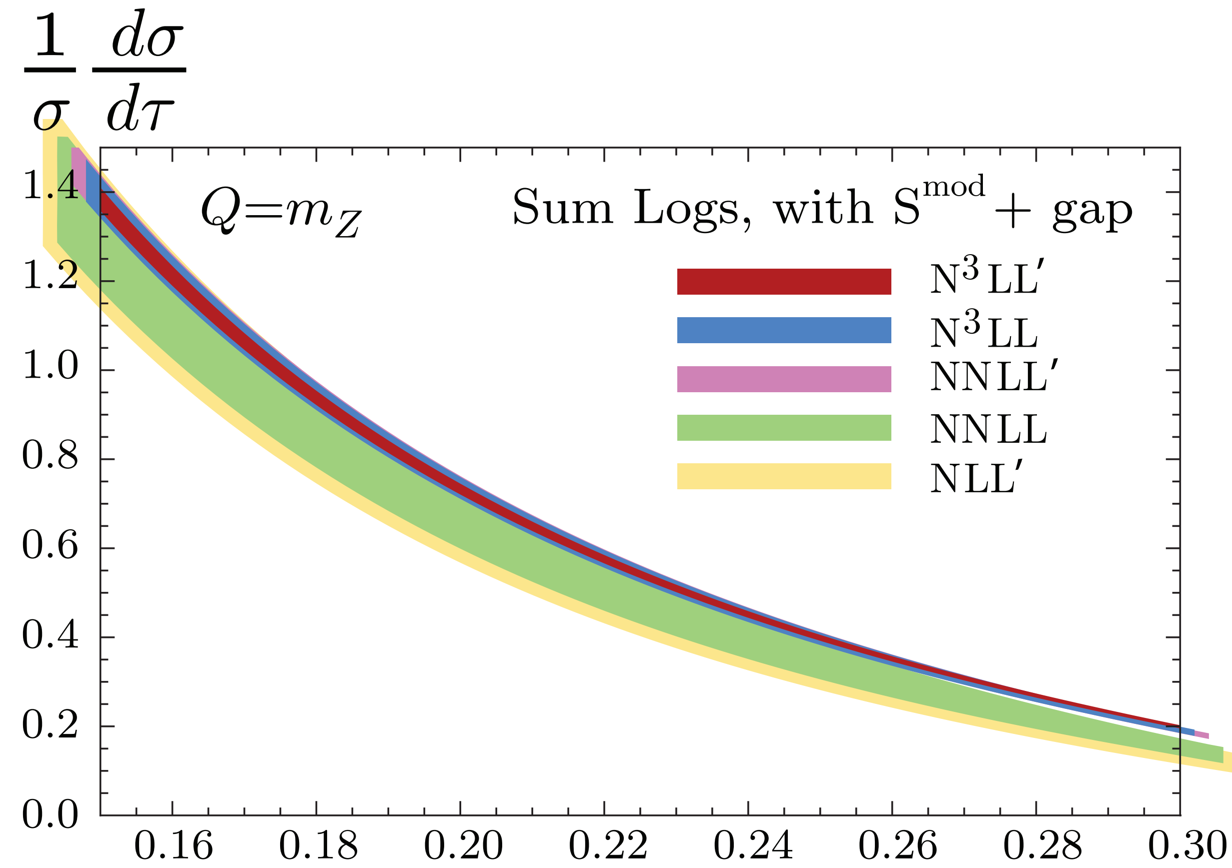
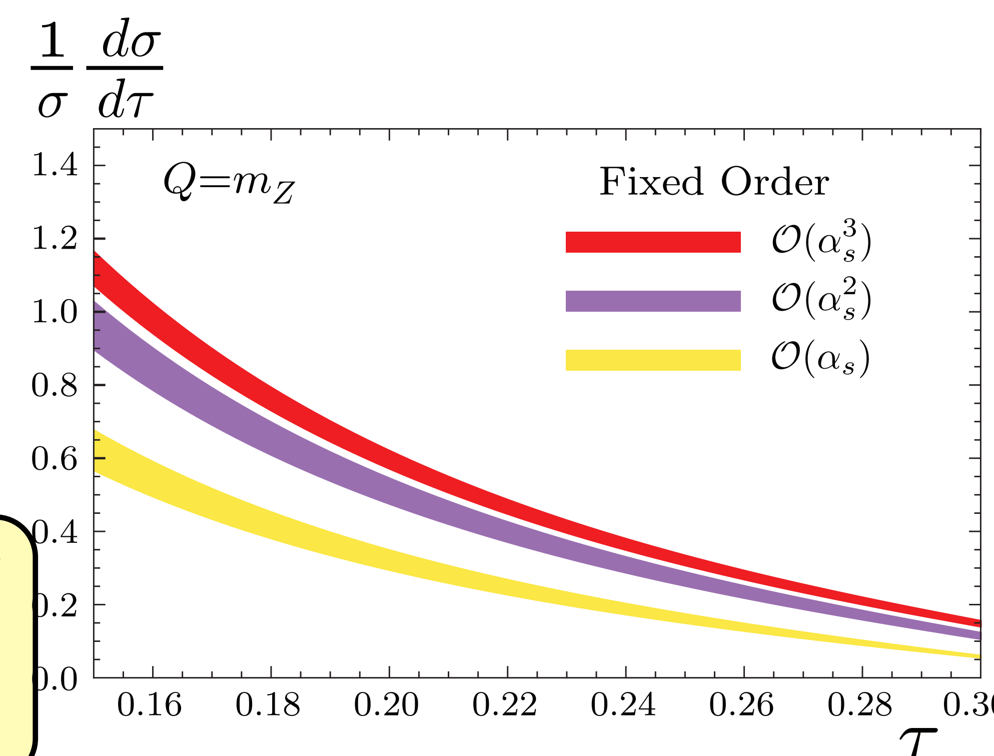
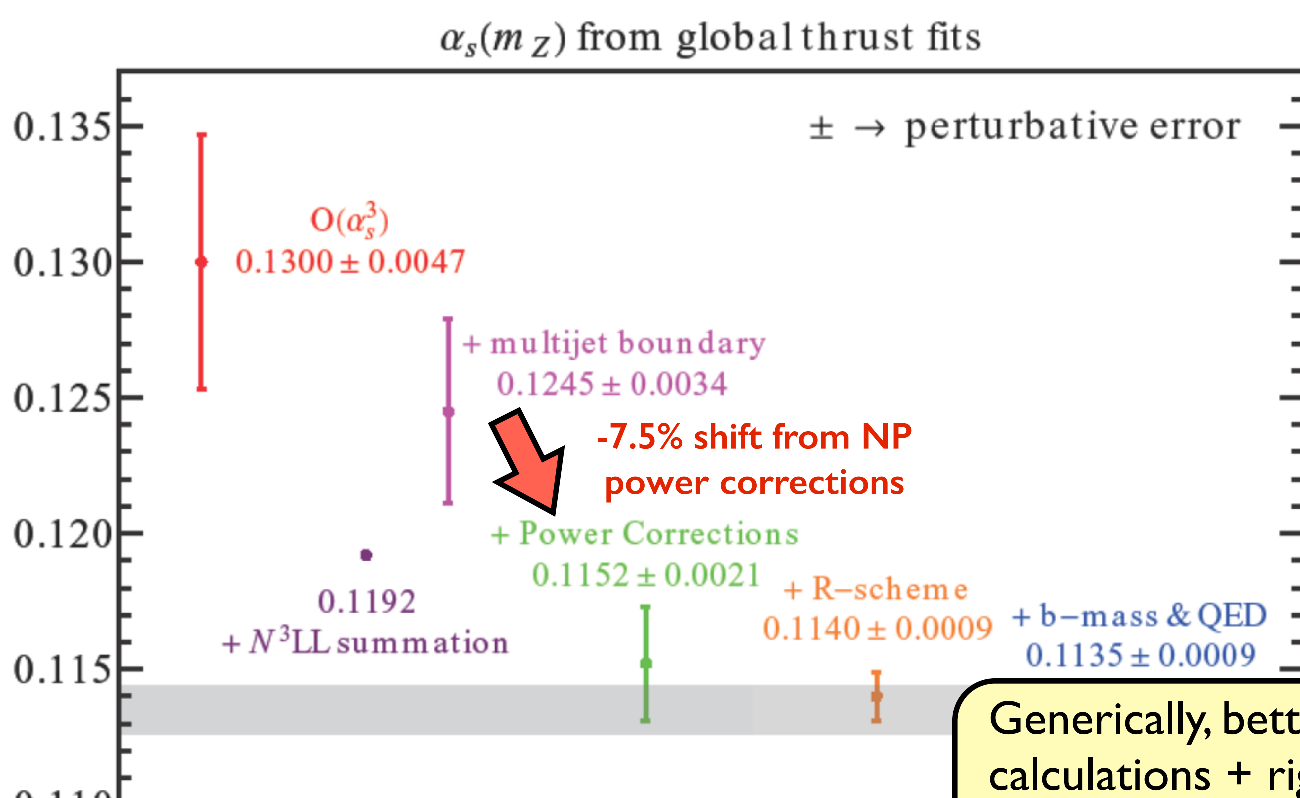
NNNLL resummed perturbative distribution

Becher, Schwartz (2008)



Compare fixed order:





Abbate, Fickinger, Hoang,

Generically, better perturbative calculations + rigorous treatment of nonperturbative corrections gives smaller α_s

Theoretical Uncertainty

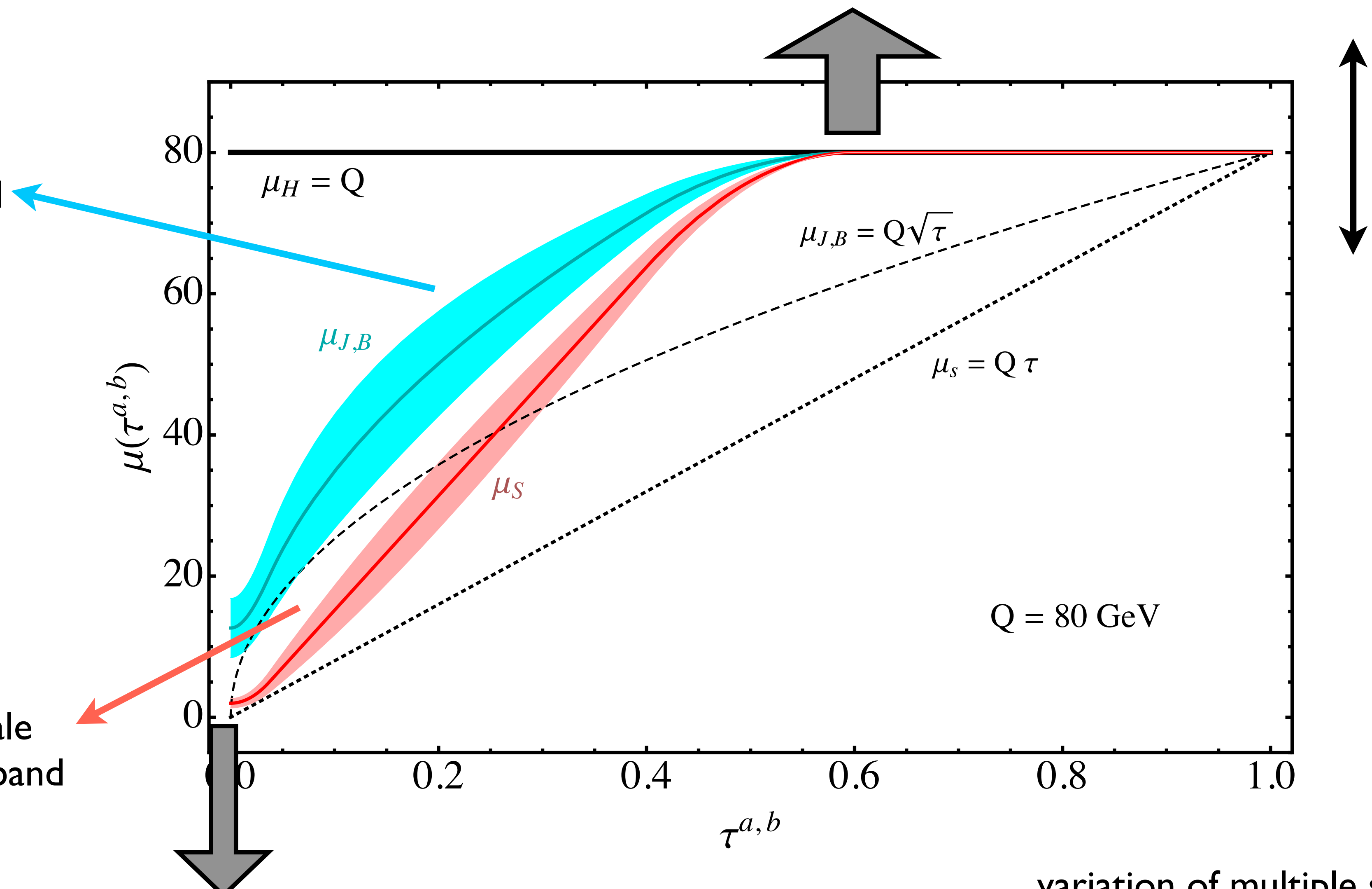
Uncertainty from missing terms of higher order in perturbation theory estimated by varying the scales in three ways:

Stewart, Tackmann, Waalewijn (2010)

need to turn off resummation for matching to fixed order at large τ

2. jet/beam scale within cyan band

1. overall scale between Q/2 and 2Q

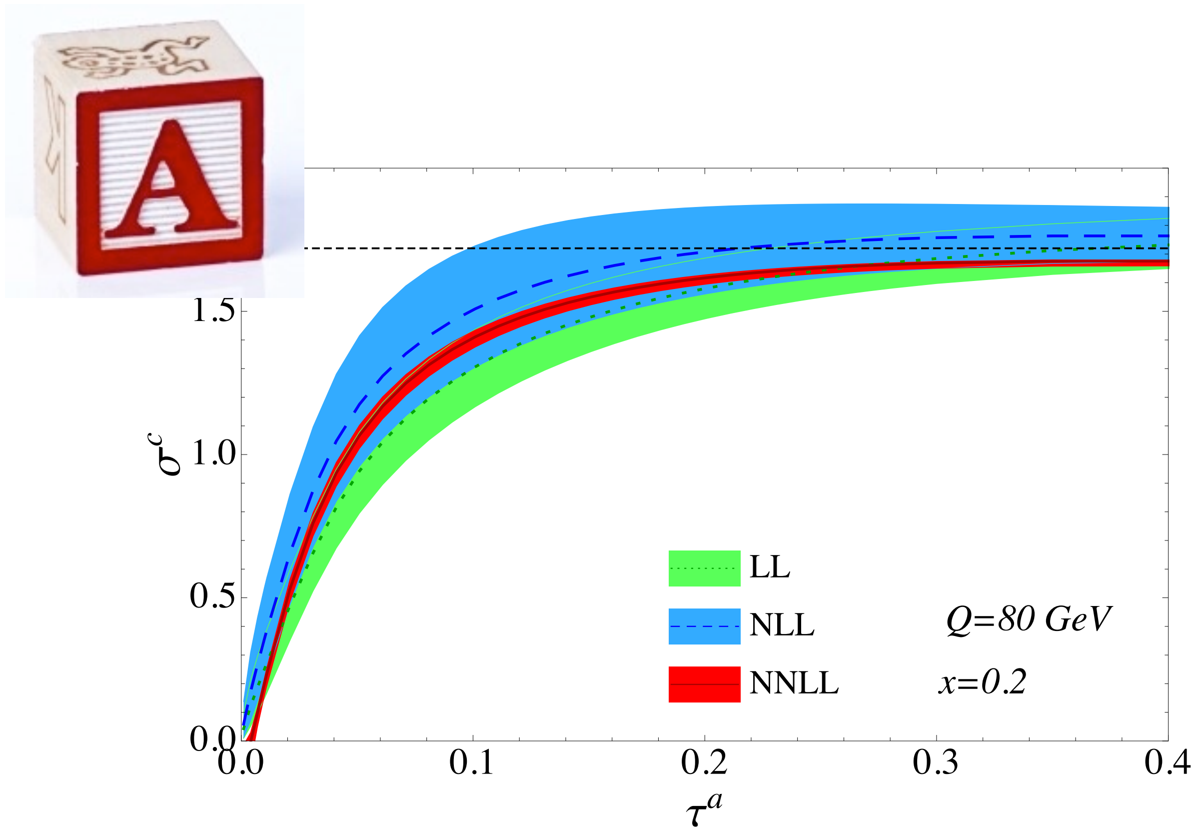


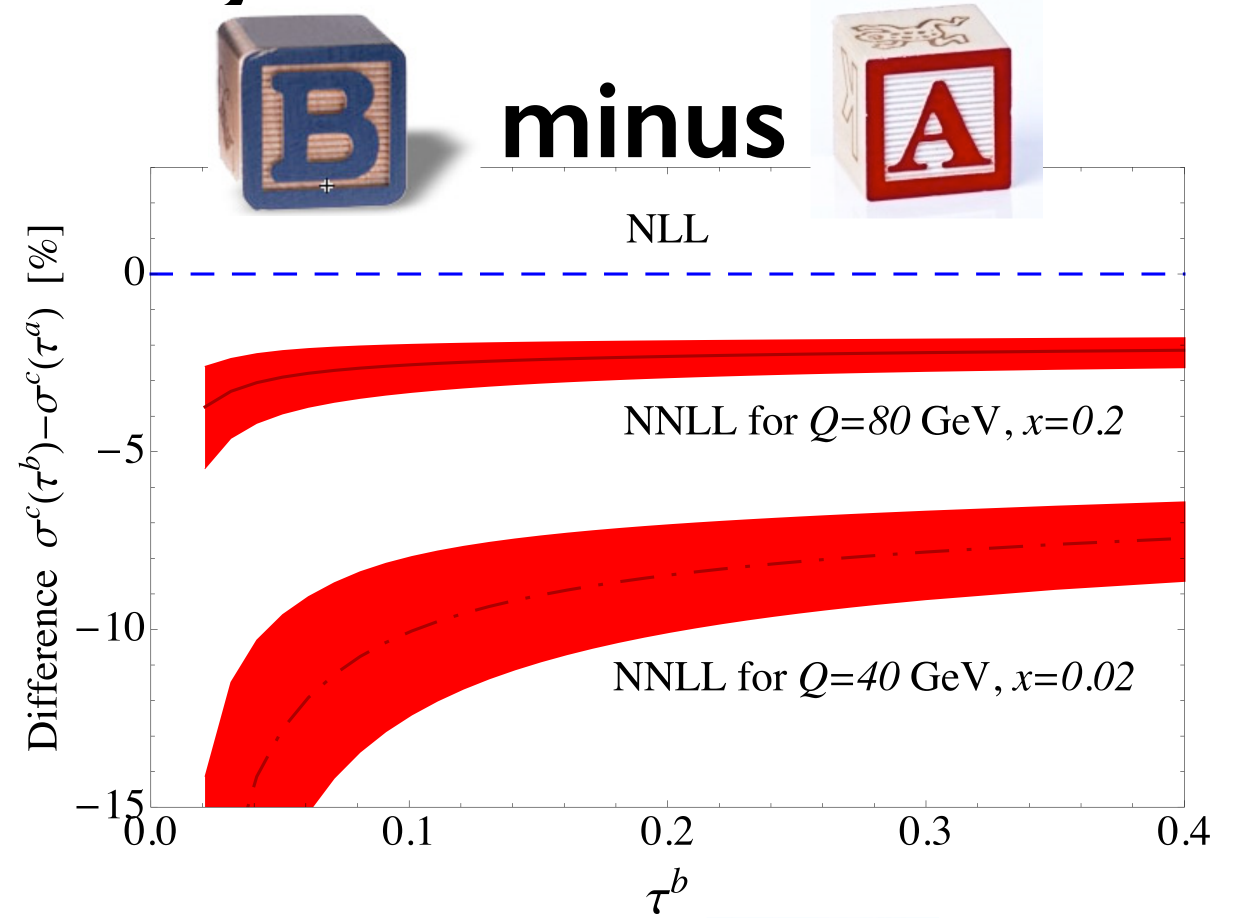
3. soft scale within red band

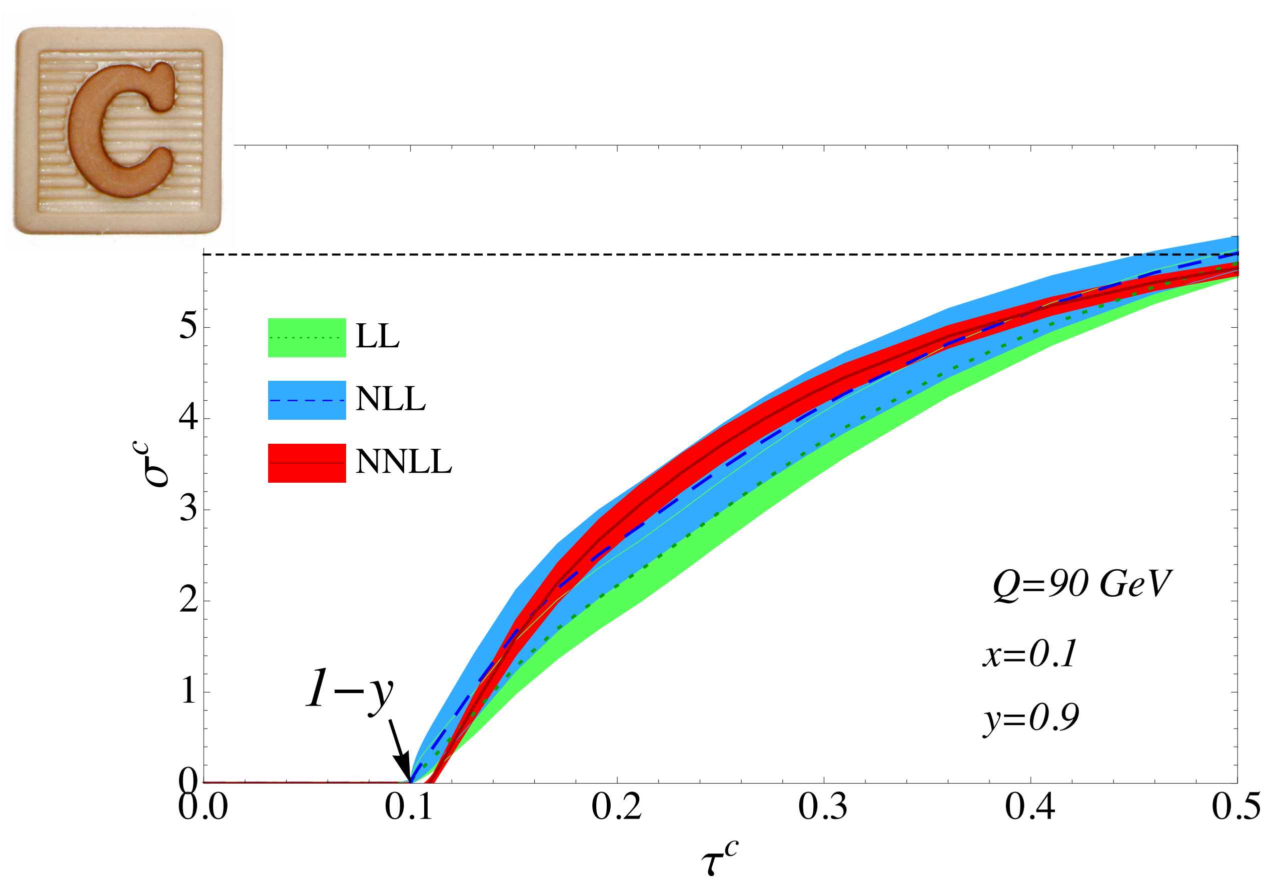
nonzero minimum values of scales at small τ

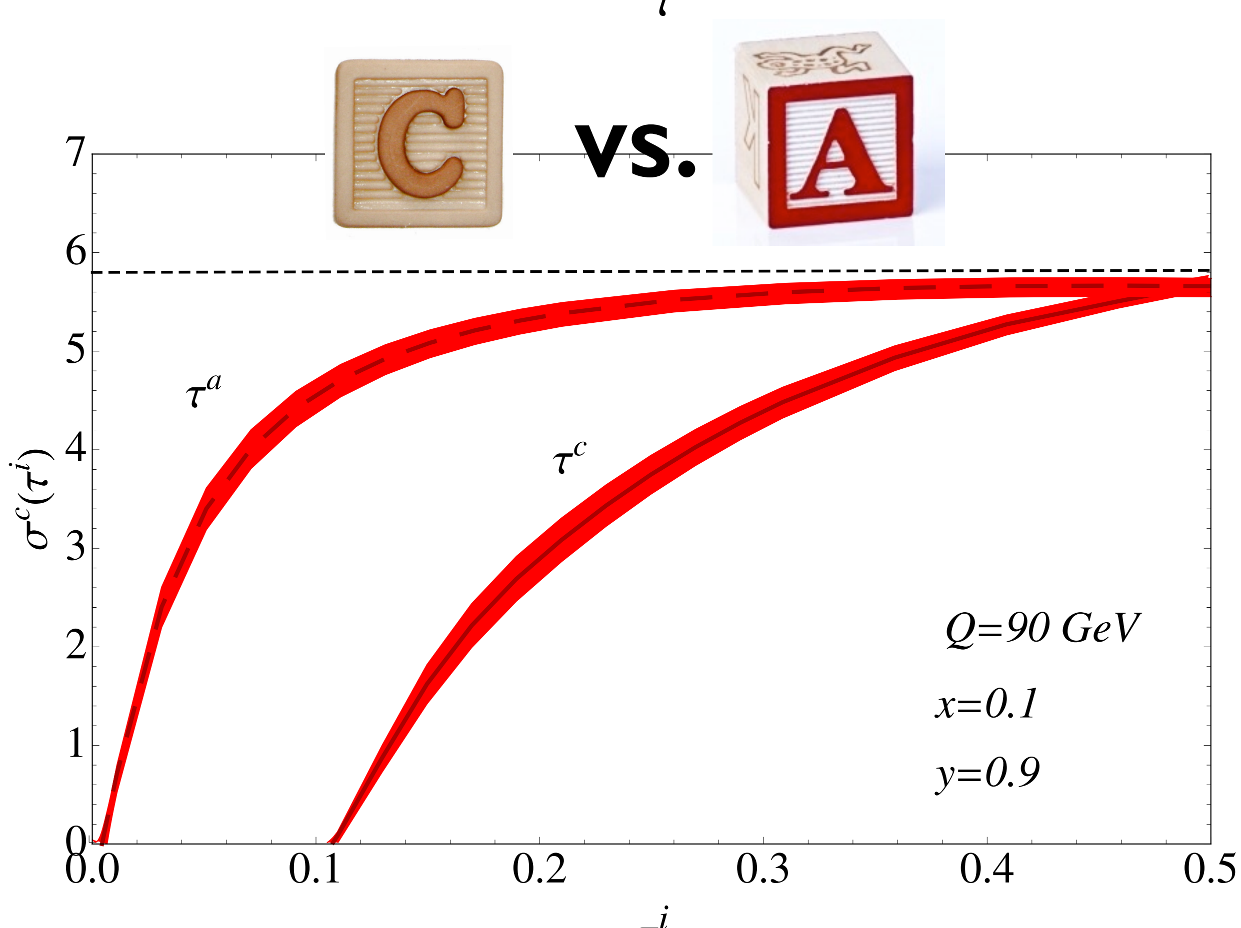
variation of multiple scales allows more realistic estimate of uncertainty than fixed-order or traditional pQCD predictions which only allow one (overall) scale variation

Predictions for DIS 1-jettiness

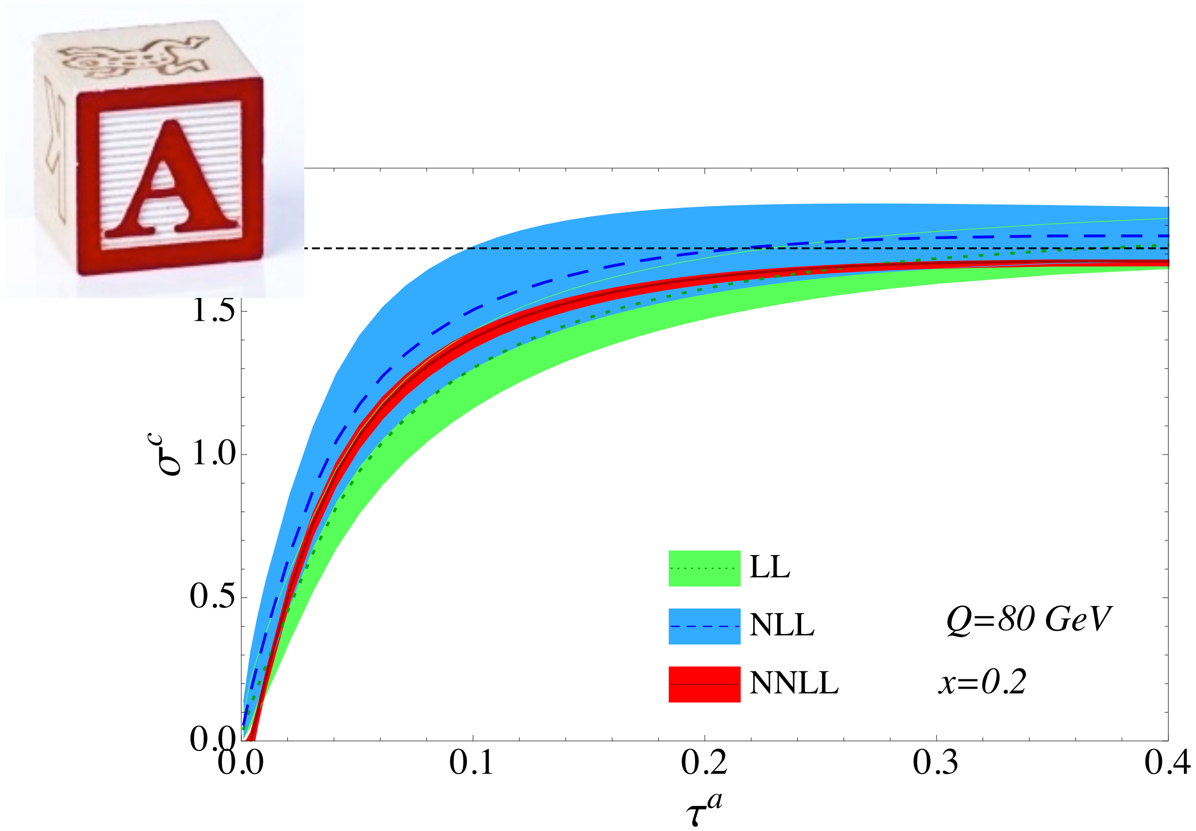


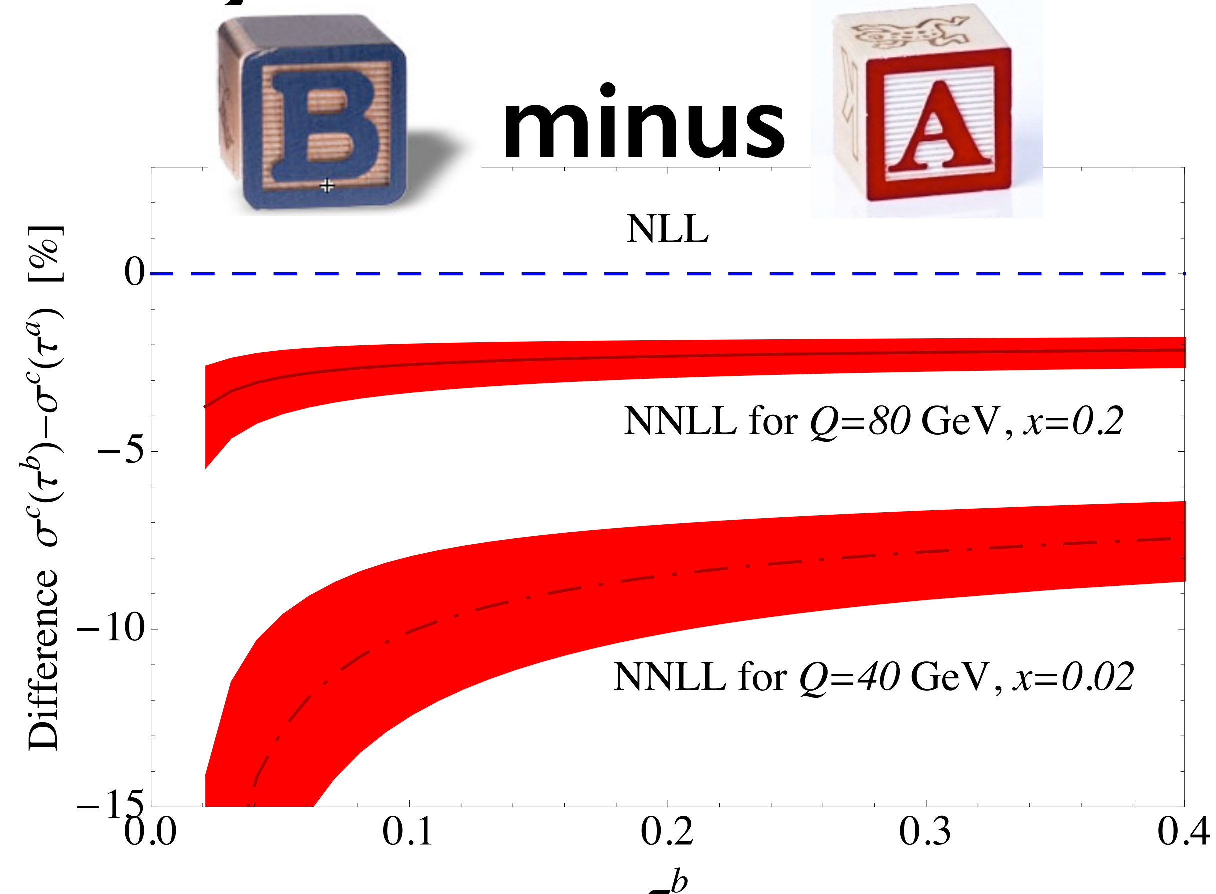


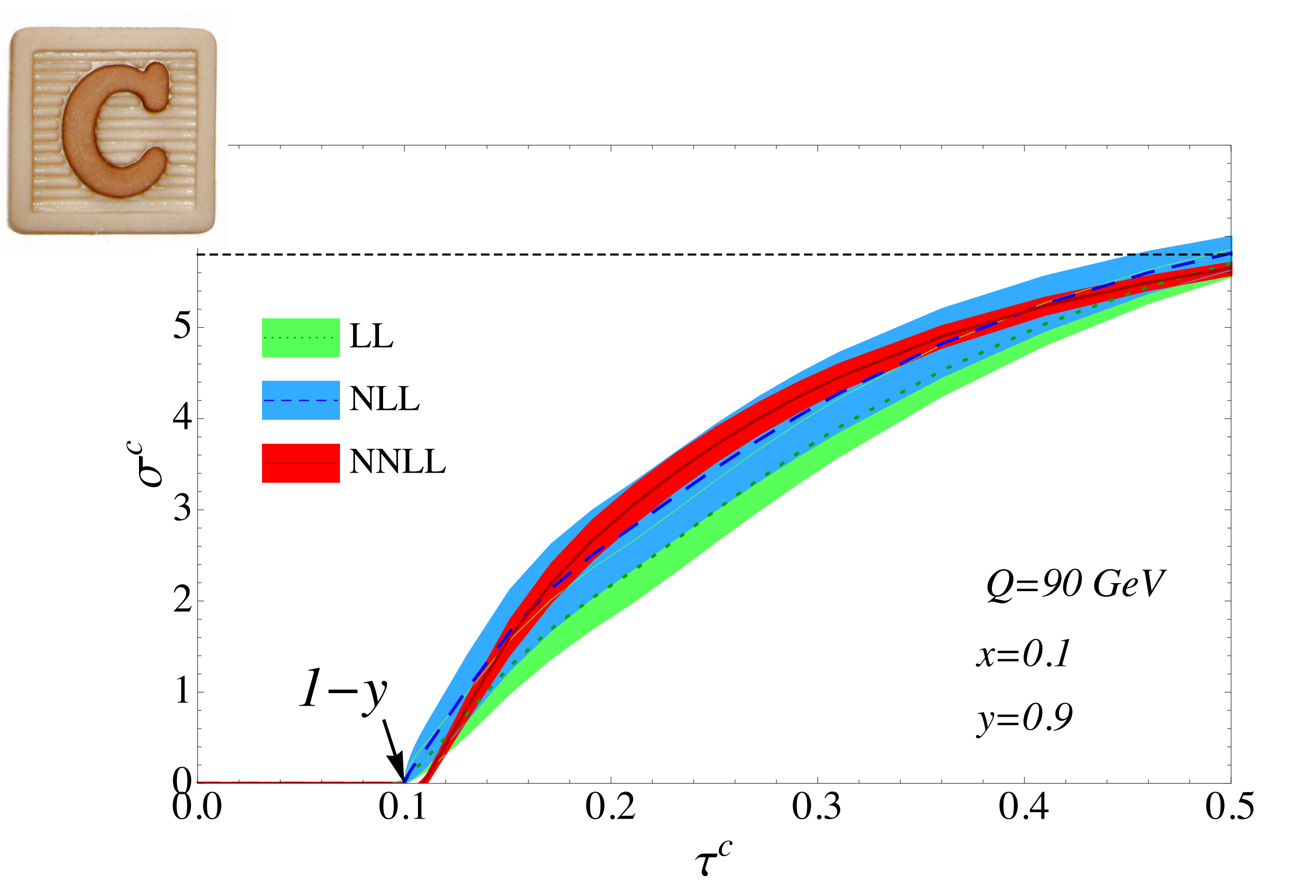


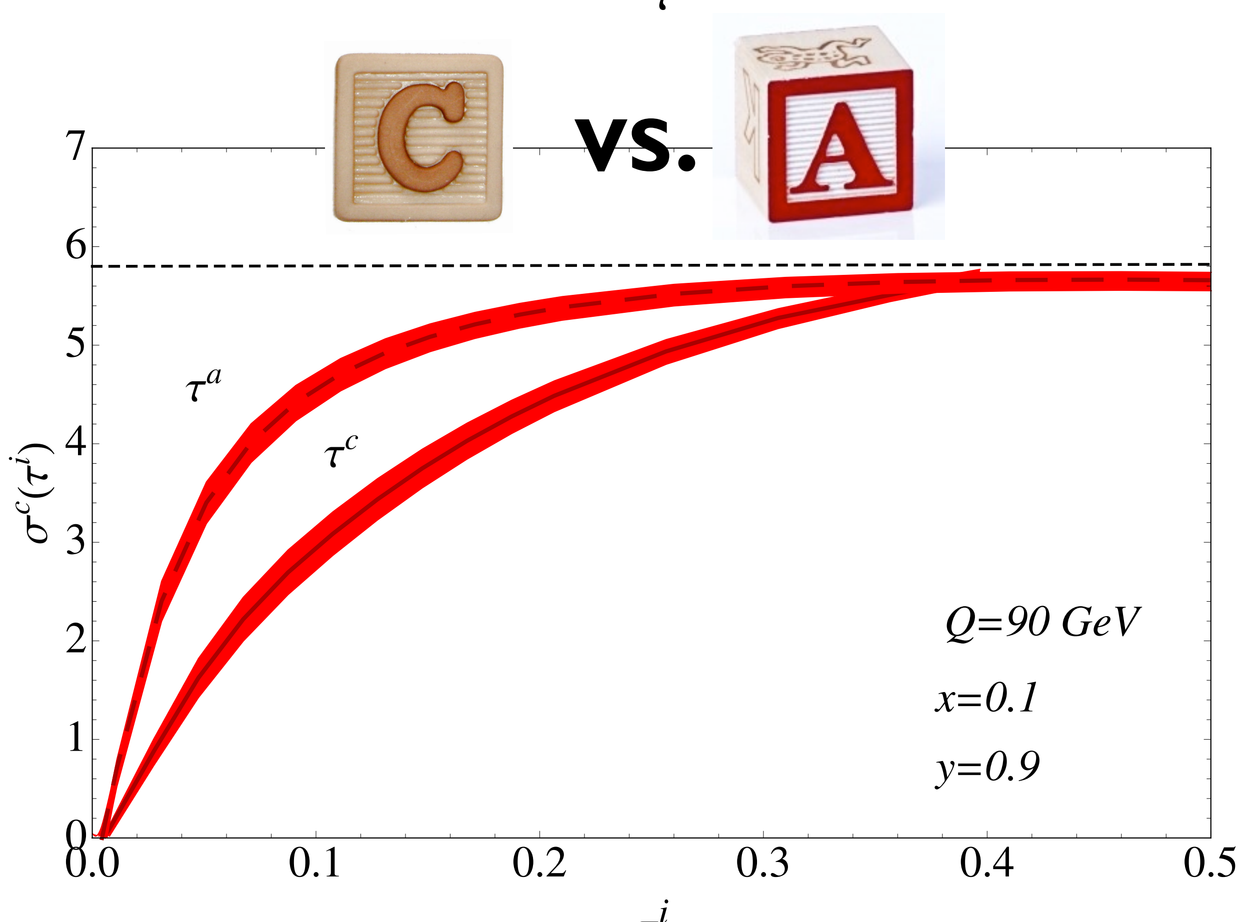


Predictions for DIS 1-jettiness



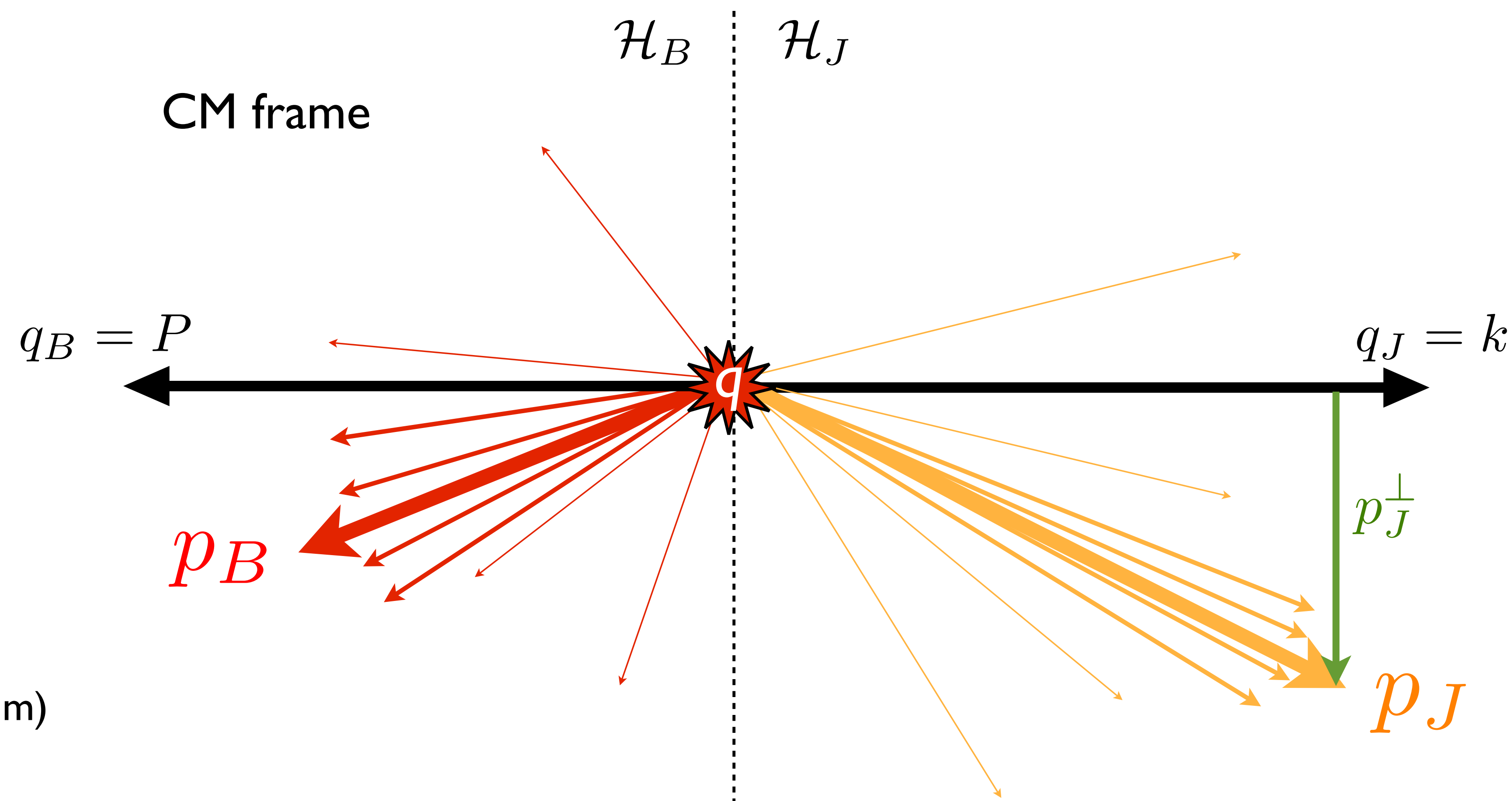






Three choices for DIS 1-jettiness

τ_1^c



$$q_B = P$$

$$q_J = k$$

(electron momentum)

measures thrust in back-to-back hemispheres in Center-of-momentum frame

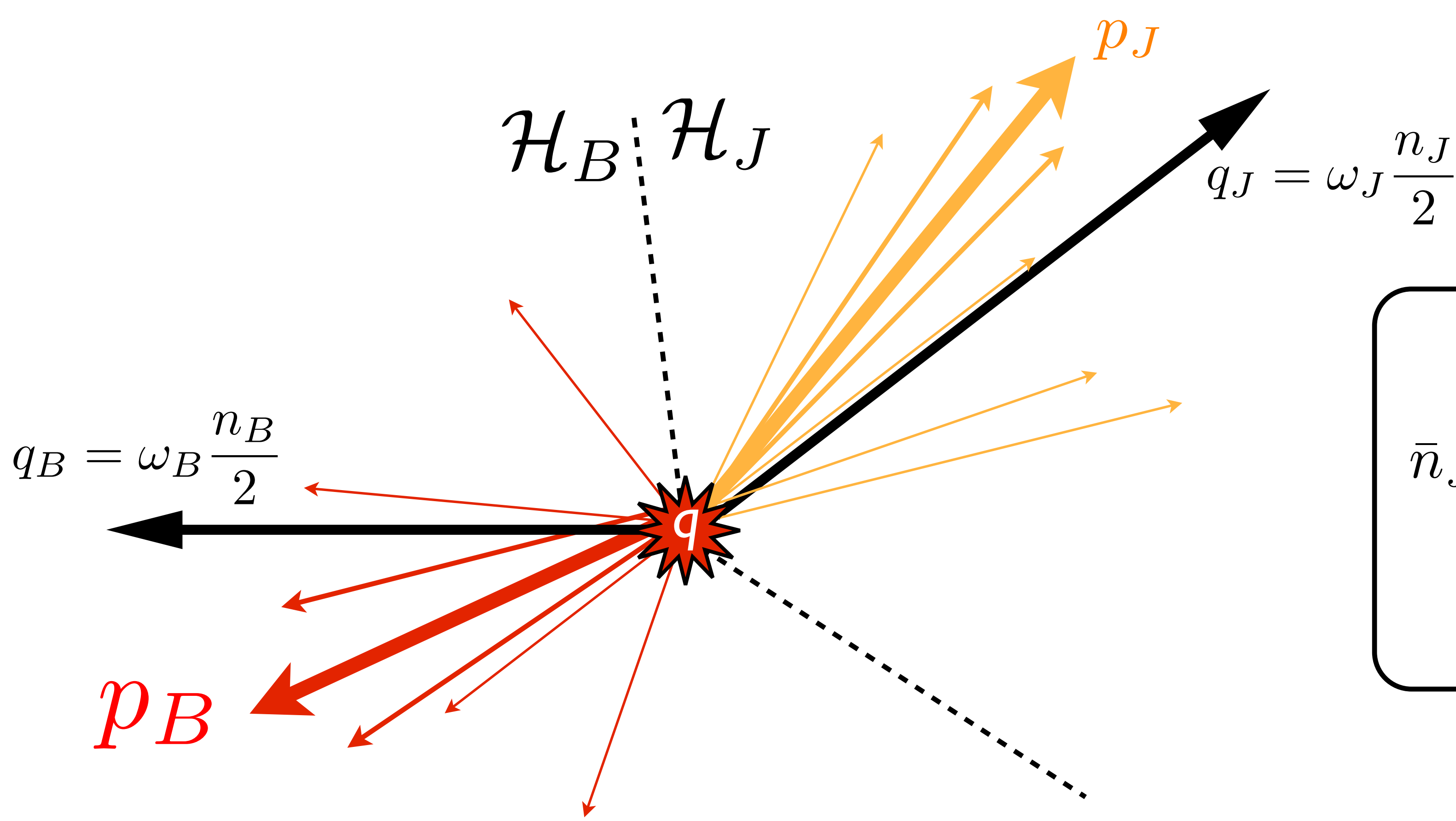
momentum transfer ***q*** itself has a nonzero transverse component:

$$q = y\sqrt{s}\frac{n_z}{2} - xy\sqrt{s}\frac{\bar{n}_z}{2} + \sqrt{1-y}\,Q\hat{n}_\perp$$

seemingly simplest definition: *in practice* hardest to calculate!

Restriction: $p_J^\perp$ has to be small for 1-jettiness τ_1^c to be small $\Rightarrow 1 - y \sim \lambda^2$

Light-Cone Directions



Choose conjugate directions:

$$\bar{n}_J = \frac{2}{n_J \cdot n_B} n_B \qquad \bar{n}_B = \frac{2}{n_J \cdot n_B} n_J$$

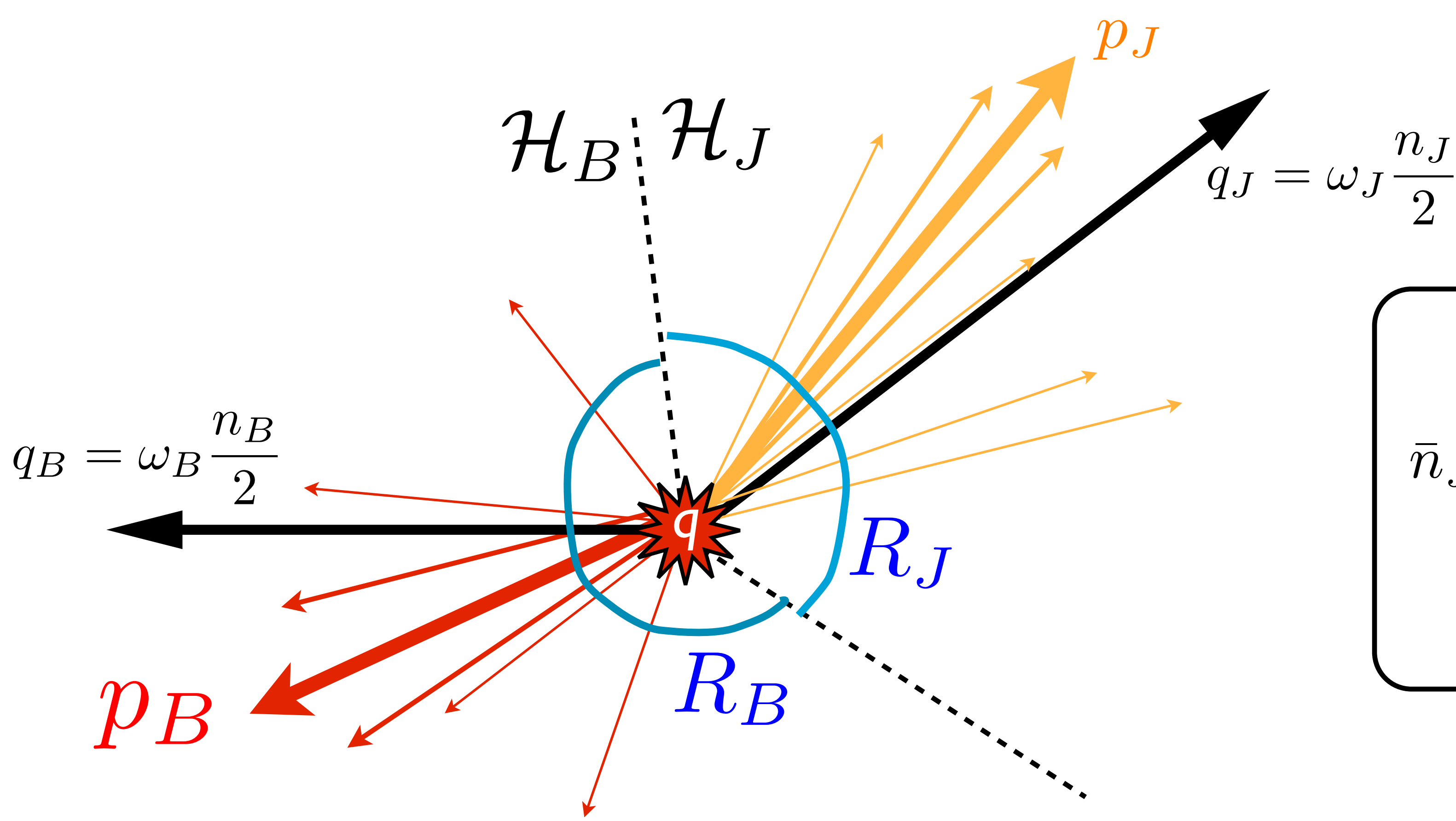
$$\Rightarrow n_J \cdot \bar{n}_J = n_B \cdot \bar{n}_B = 2$$

Sizes of beam and jet regions:

$$\frac{n_J \cdot p}{\bar{n}_J \cdot p} < \frac{\omega_B}{\omega_J} \frac{n_J \cdot n_B}{2} \equiv R_J^2$$

$$\frac{n_B \cdot p}{\bar{n}_B \cdot p} < \frac{\omega_J}{\omega_B} \frac{n_J \cdot n_B}{2} \equiv R_B^2$$

Light-Cone Directions



Choose conjugate directions:

$$\bar{n}_J = \frac{2}{n_J \cdot n_B} n_B \qquad \bar{n}_B = \frac{2}{n_J \cdot n_B} n_J$$

$$\Rightarrow n_J \cdot \bar{n}_J = n_B \cdot \bar{n}_B = 2$$

Sizes of beam and jet regions:

$$\frac{n_J \cdot p}{\bar{n}_J \cdot p} < \frac{\omega_B}{\omega_J} \frac{n_J \cdot n_B}{2} \equiv R_J^2$$

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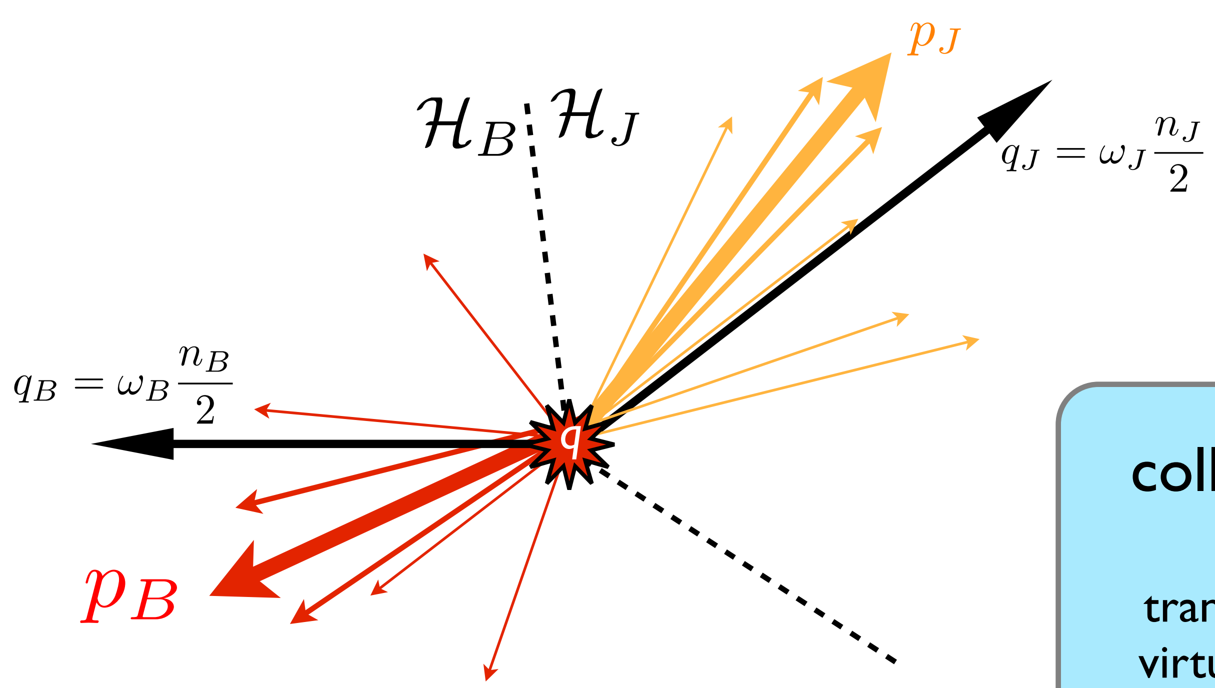
Beam, Jet, and Soft Contributions



In each case of 1-jettiness, $\tau_1 = \frac{n_J \cdot p_J}{Q_J} + \frac{n_B \cdot p_B}{Q_B}$

contributions from different modes: $\tau_1 = \tau_J^{n_J} + \tau_B^{n_B} + \tau_S$

collinear contributions: $\tau_J^{n_J} = \frac{t_J}{s_J}\,, \quad \tau_B^{n_B} = \frac{t_B}{s_B}$

transverse virtualities: $t_J = \bar{n}_J \cdot p_J \, n_J \cdot p_J^{n_J}$ $\quad t_B = \bar{n}_B \cdot p_B \, n_B \cdot p_B^{n_B}$

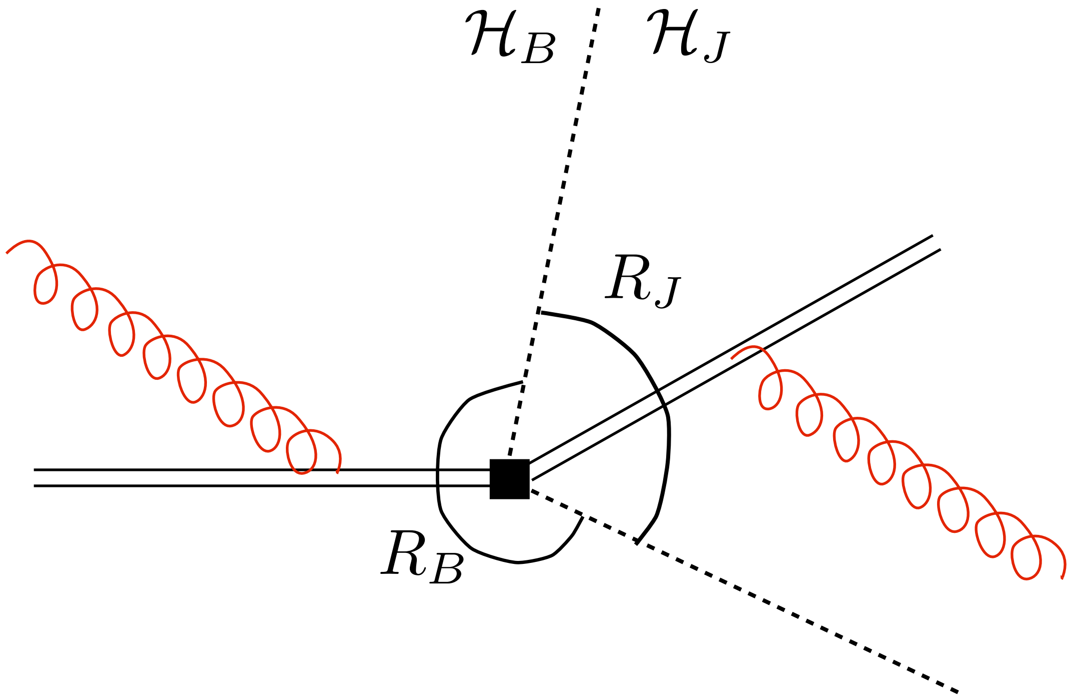
soft contribution: $\tau_S = \frac{n_J \cdot k_J}{Q_J} + \frac{n_B \cdot k_B}{Q_B} \longrightarrow \frac{n_J' \cdot k_J + n_B' \cdot k_B}{Q_R}$

soft boost invariance: $n_{J,B} \longrightarrow n_{J,B}' = \frac{n_{J,B}}{R_{J,B}}$ $\quad Q_R \equiv \frac{Q_J}{R_J} = \frac{Q_B}{R_B}$

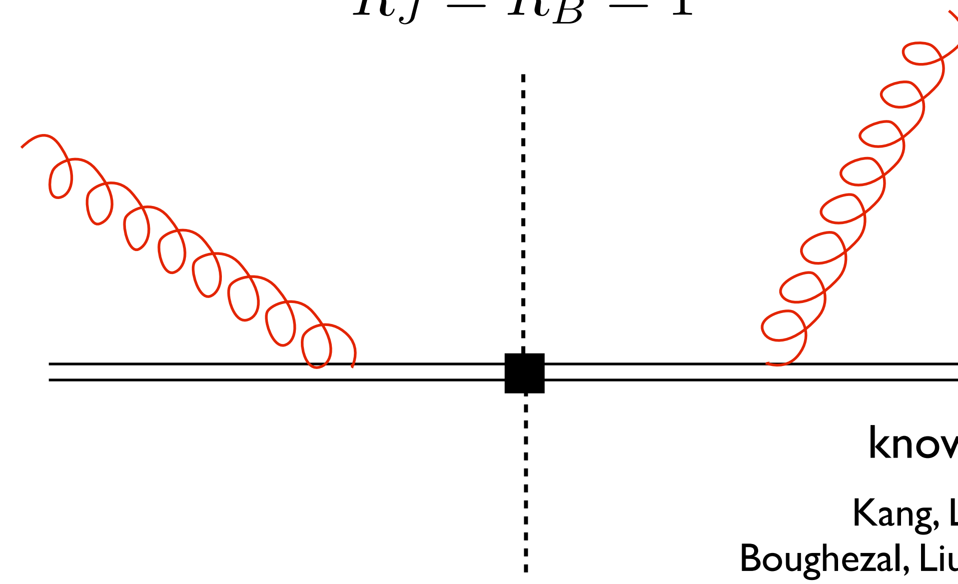
Lorentz invariant constants:

	s_J	s_B	Q_R
τ_1^a	Q^2	Q^2	Q
τ_1^b	Q^2	Q^2	Q
τ_1^c	Q^2	xQ^2	$\sqrt{x}\,Q$

Boost to Hemisphere Soft Function



1-jettiness soft function



hemisphere soft function

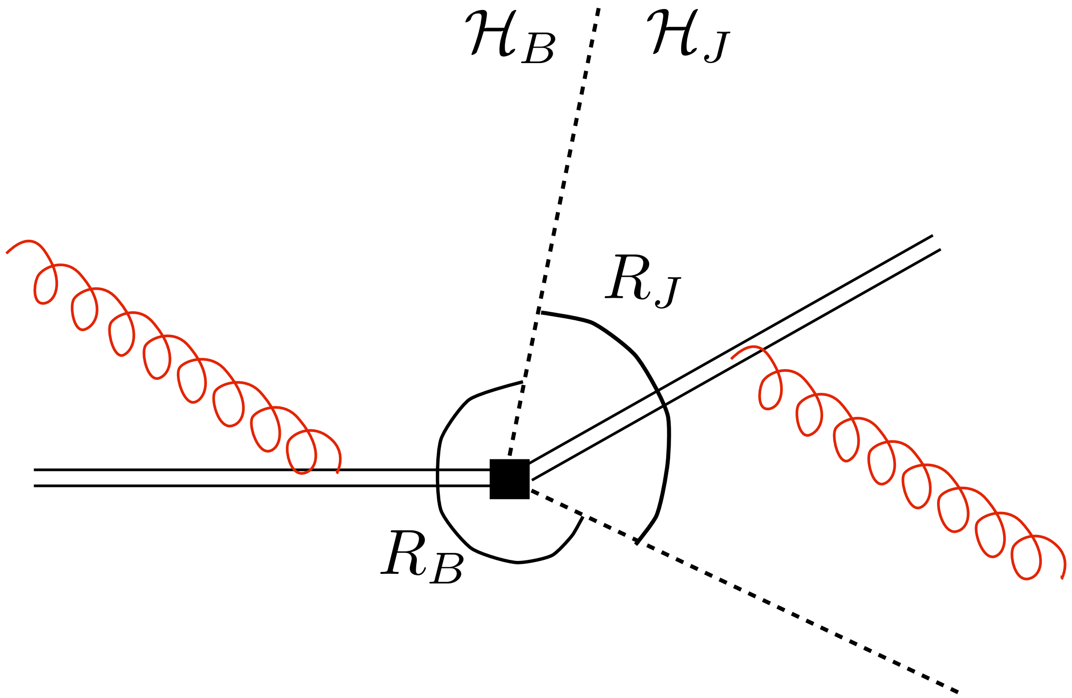
known to 2 loops

Kang, Labun, CL (2015); Boughezal, Liu, Petriello (2015)

anomalous dimension known to 3 loops

$$S(k_J, k_B, q_J, q_B, \mu) = \frac{1}{N_C}\,\mathrm{tr}\sum_{X_s}\left|\langle X_s|\,T[Y^\dagger_{n_B} Y_{n_J}](0)\,|0\rangle\right|^2 \delta\Big(k_J - \sum_{i\in X_s}\theta(q_B\cdot k_i - q_J\cdot k_i)n_J\cdot k_i\Big)$$
$$\times\,\delta\Big(k_B - \sum_{i\in X_s}\theta(q_J\cdot k_i - q_B\cdot k_i)n_B\cdot k_i\Big)$$

Boost to Hemisphere Soft Function



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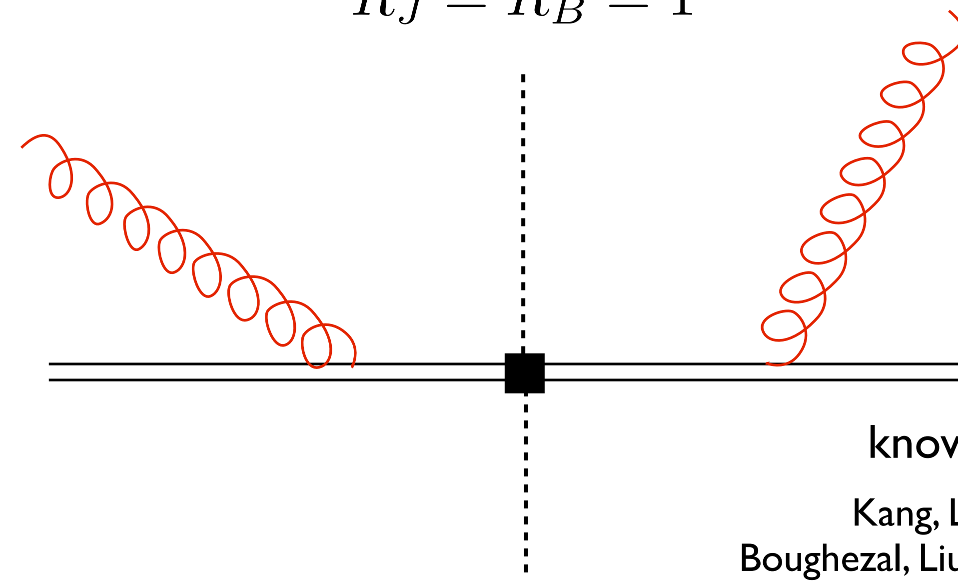

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$$\times \, \delta\Big(k_B - \sum_{i \in X_s} \theta(q_J \cdot k_i - q_B \cdot k_i) n_B \cdot k_i \Big)$$

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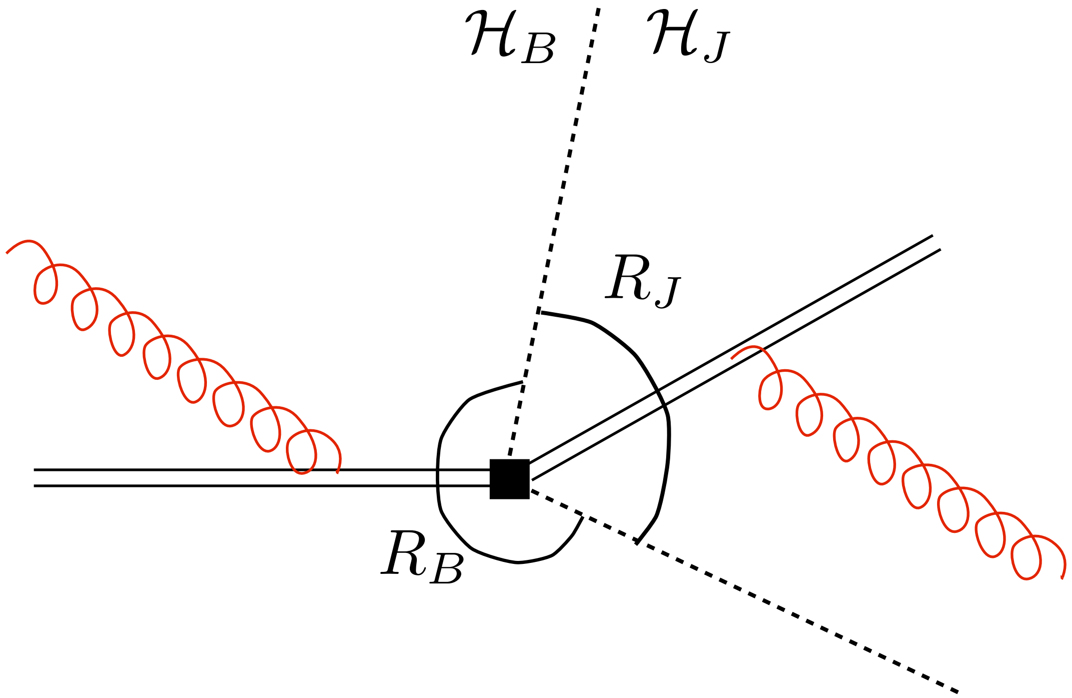
Boost to Hemisphere Soft Function



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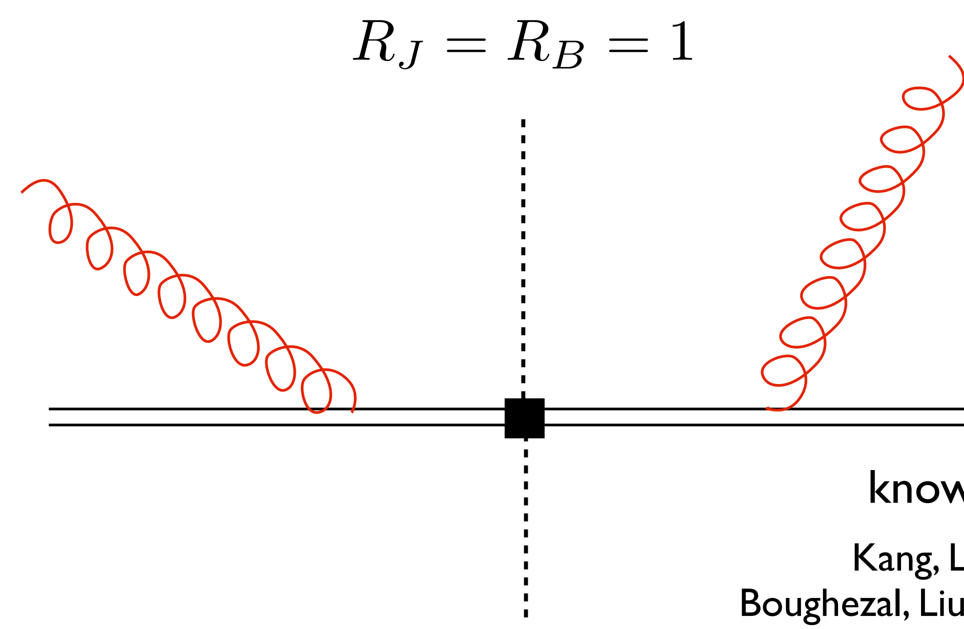

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$$\times \, \delta\Big(\frac{k_B}{R_B} - \sum_{i \in X_s} \theta(n'_J \cdot k_i - n'_B \cdot k_i) n'_B \cdot k_i \Big) = \frac{1}{R_J R_B} S_{\mathrm{hemi}}\Big(\frac{k_J}{R_J}, \frac{k_B}{R_B}, \mu \Big) \,.$$

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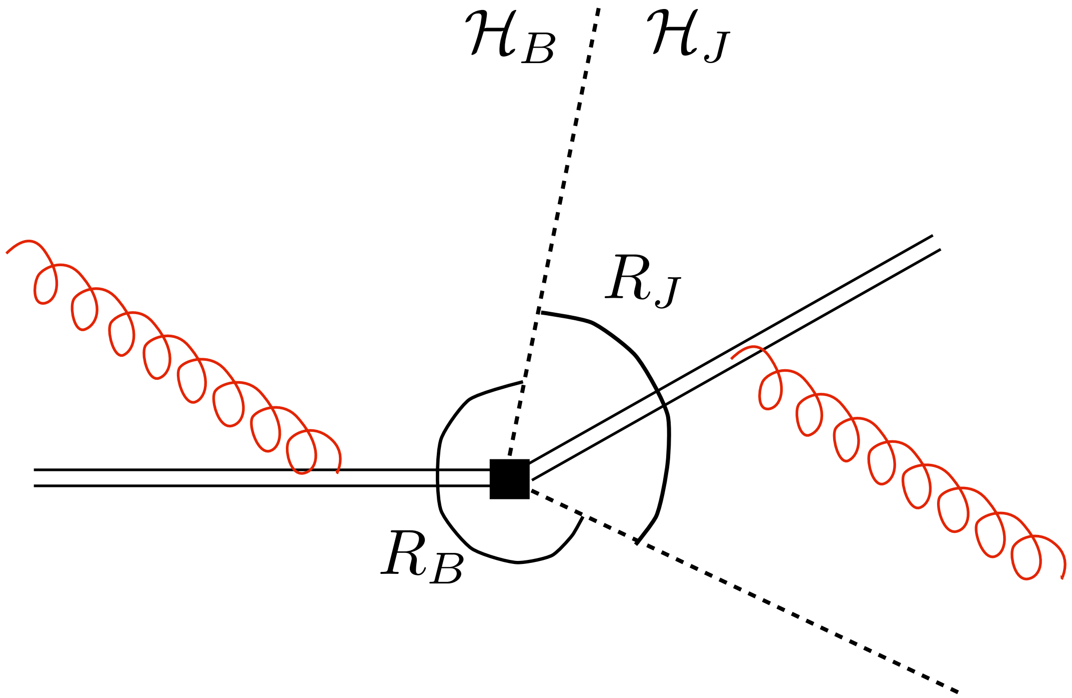
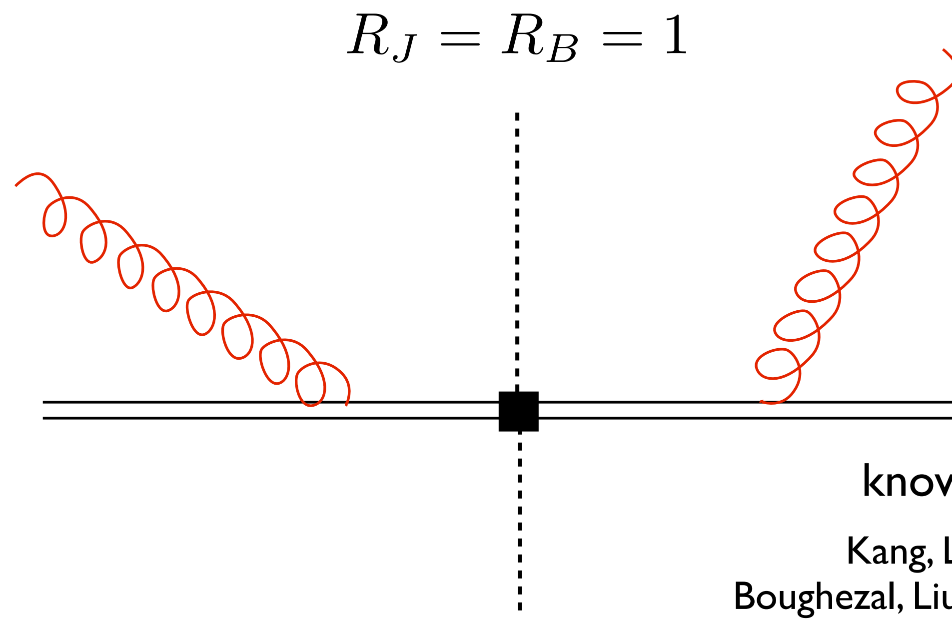
Boost to Hemisphere Soft Function



1-jettiness soft function



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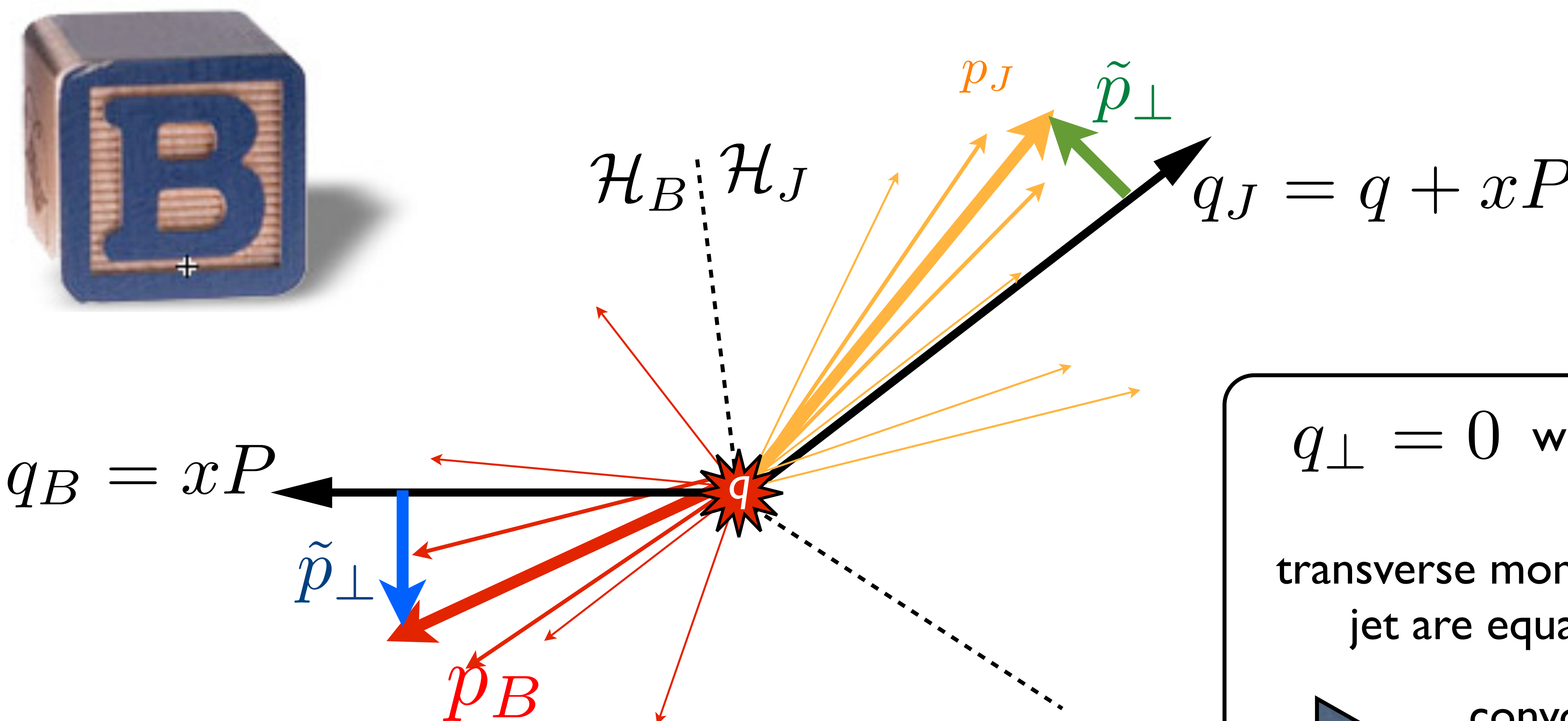
$$S(k_J, k_B, R_J, R_B, \mu) = \frac{1}{R_J R_B} S_{\mathrm{hemi}}\left(\frac{k_J}{R_J}, \frac{k_B}{R_B}, \mu \right)$$

cf. Feige, Schwartz, Stewart, Thaler (2012)

Transverse jet and beam momenta

Convolution between jet and beam transverse momenta:

$$W \supset \boxed{\int d^2\tilde{p}_\perp} \langle P_{n_B}|\bar{\chi}_{n_B}(0)\delta(Q_B\tau_B - n_B\cdot\hat{p}^{n_B})[\delta(\bar{n}_B\cdot q + \bar{n}_B\cdot\mathcal{P})\boxed{\delta^2(\tilde{p}_\perp - \mathcal{P}_\perp)}\chi_{n_B}](0)|P_{n_B}\rangle$$
$$\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\boxed{\delta^2(q_\perp + \tilde{p}_\perp + \mathcal{P}_\perp)}\bar{\chi}_{n_J}(0)|0\rangle$$



$q_\perp = 0$ with respect to this $\boldsymbol{q_J}$ and $\boldsymbol{q_B}$

transverse momenta of beam and jet are equal and balanced

convolution over $\tilde{p}_\perp$ remains, depends on kT-dependent beam function (perturbative kT)

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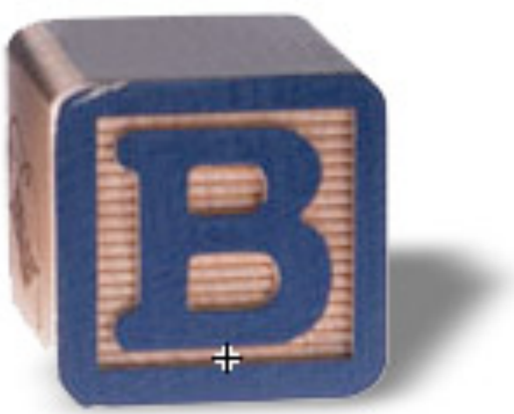

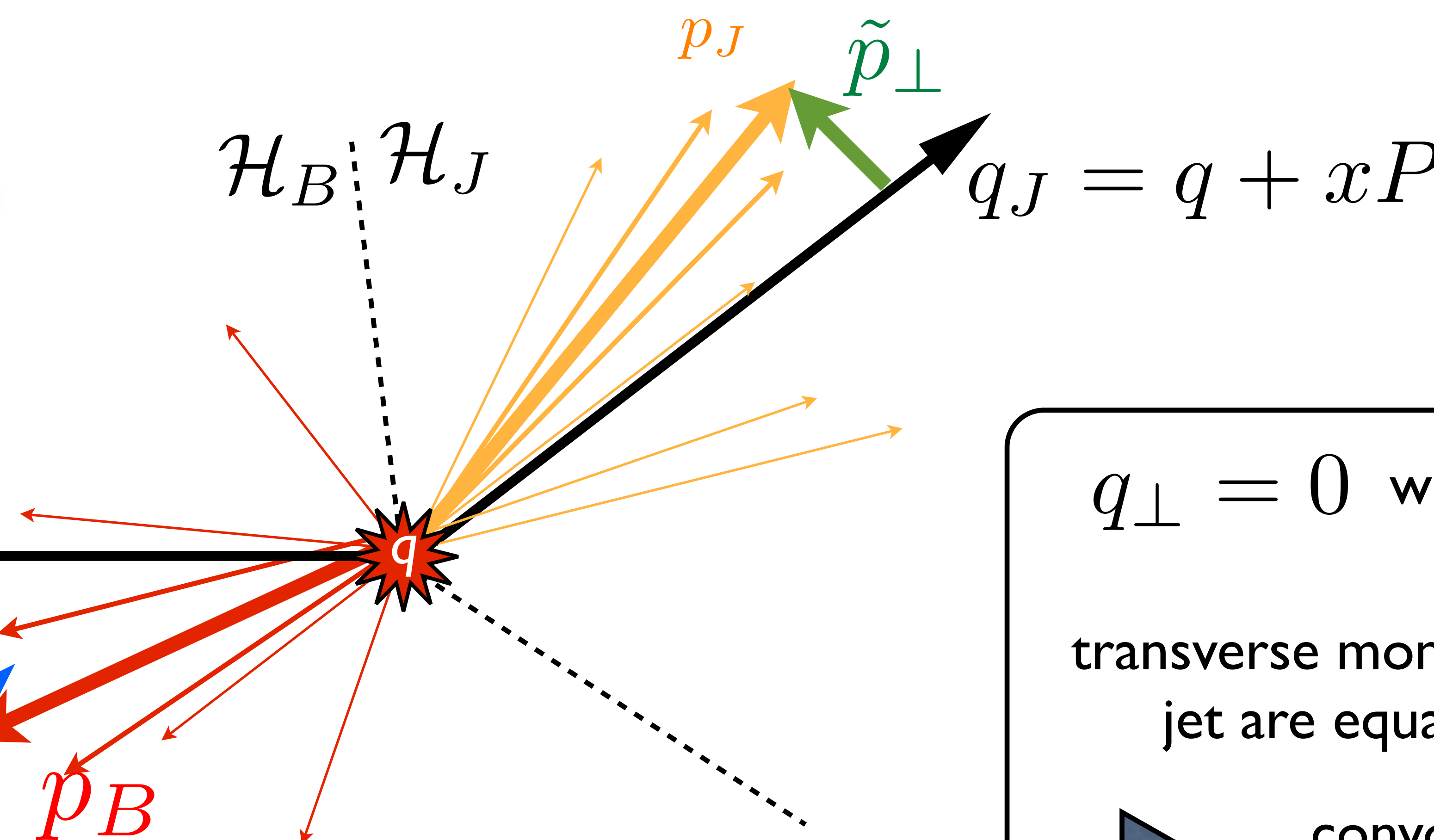

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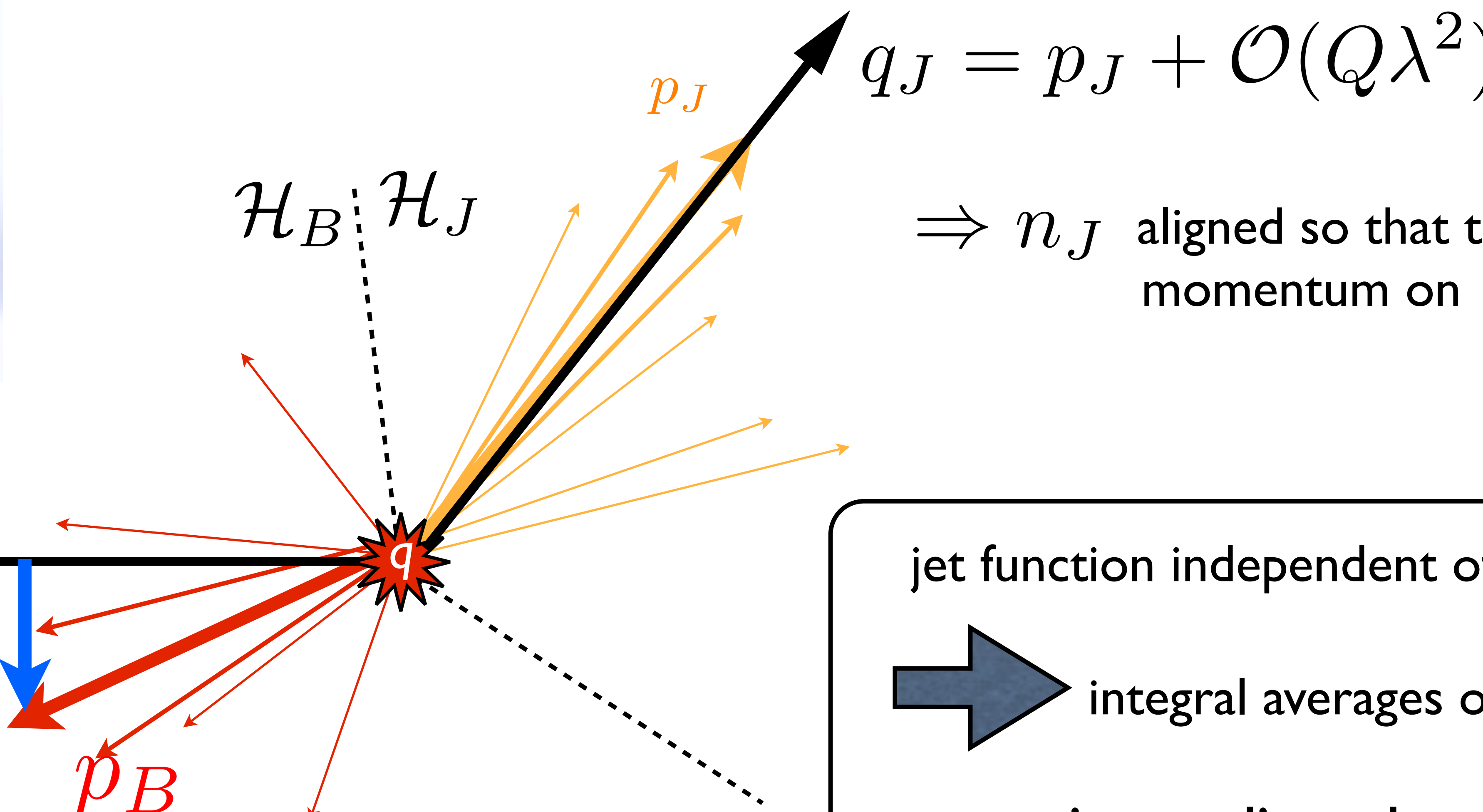

$\Rightarrow n_J$ aligned so that transverse label momentum on χ_{n_J} is zero

jet function independent of beam $\tilde{p}_\perp$

integral averages over beam $\tilde{p}_\perp$

turns into ordinary beam function

difference between q_J axes for case A and B is a leading-order effect on the argument of beam and jet functions

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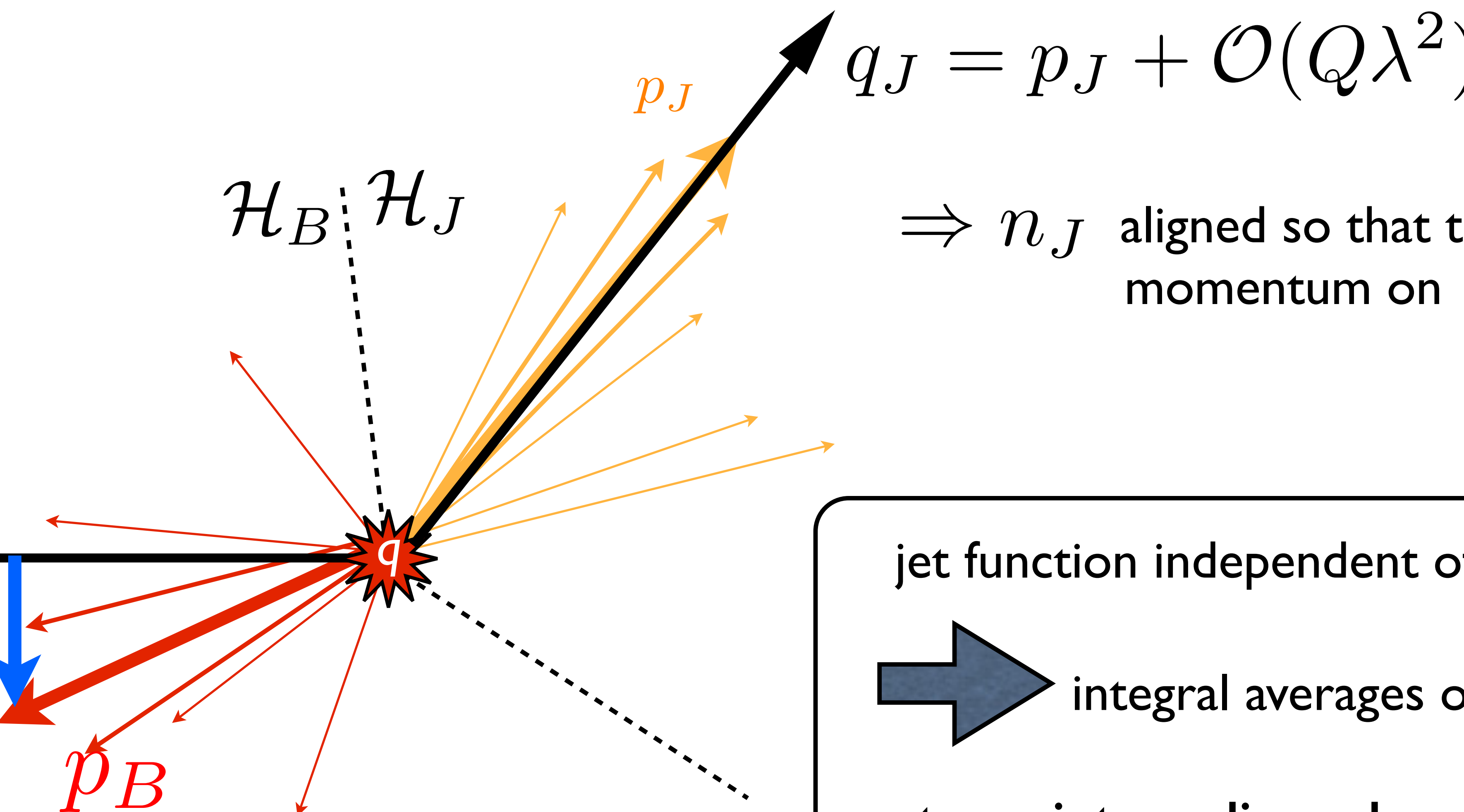
$$\times \langle 0|\chi_{n_J}(0)\delta(Q_J\tau_J - n_J\cdot\hat{p}^{n_J})\delta(\bar{n}_J\cdot q + \bar{n}_J\cdot\mathcal{P})\delta^2(\cancel{q_\perp + \tilde{p}_\perp} + \mathcal{P}_\perp)\bar{\chi}_{n_J}(0)|0\rangle$$



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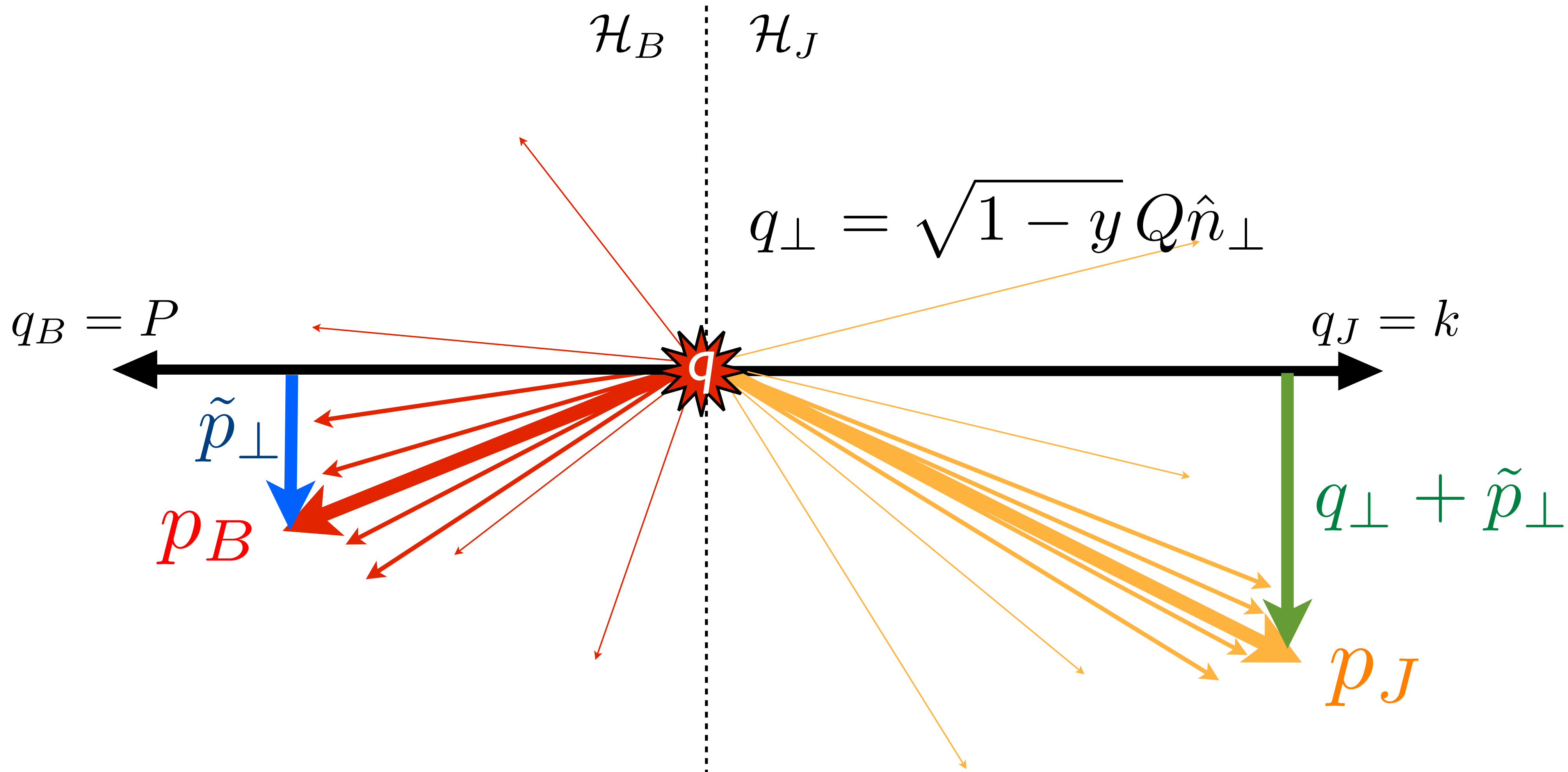
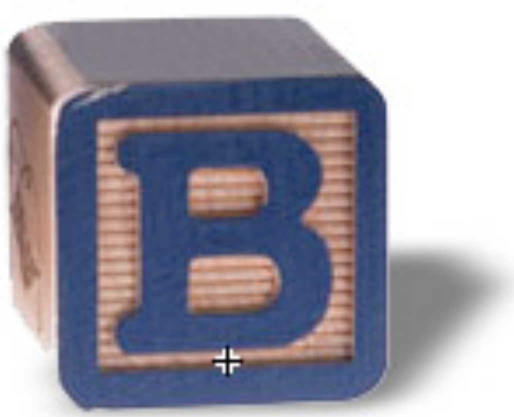


momentum transfer ***q*** itself has nonzero transverse component relative to ***P, k*** → nontrivial convolution between jet function and pT-dependent beam function

Differences between versions A and B

$$\boxed{n_J^A = n_J^B + \mathcal{O}(\lambda)}$$

$$\frac{1}{\sigma_0}\frac{d\sigma(x,Q^2)}{d\tau_1^a} = H(Q^2,\mu)\int dt_J dt_B dk_S \delta\Big(\tau_1^a - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\Big)$$
$$\times J_q(t_J,\mu) B_q(t_B,x,\mu) S(k_S,\mu)$$



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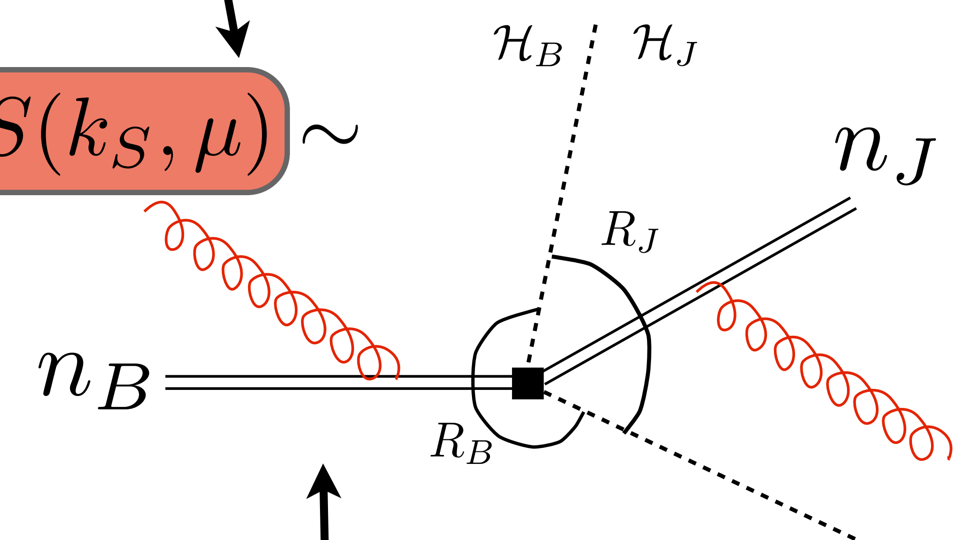
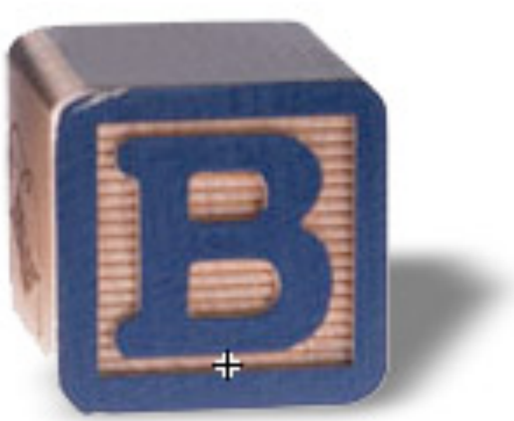
Differences power suppressed:

$$\boxed{H(Q^2,\mu) = |C(Q^2,\mu)|^2\, \mathrm{Tr}\Big(\Gamma \frac{\not n_J}{4} \Gamma' \frac{\not n_B}{4}\Big)}$$

$$\boxed{S(k_S,\mu)} \sim$$





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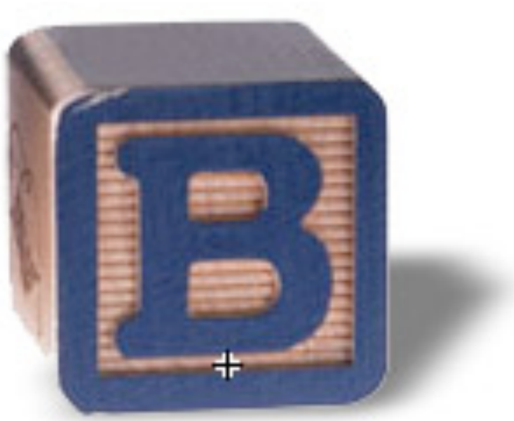
Differences
leading order:

argument of jet function

type of beam function



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